Math 2413 **Homework Set 9** **10 Points**

#2. (2 pts) $\int 12w^6 - 3w^{-4} + w^{\frac{9}{5}} - 10\,dw = \boxed{\tfrac{12}{7}w^7 + w^{-3} + \tfrac{5}{14}w^{\frac{14}{5}} - 10w + c}$

#3. (2 pts) $\int 5z^{-\frac{3}{4}} + \frac{1}{2}z^{-10} - \frac{1}{9}\frac{1}{z}\,dz = \boxed{20z^{\frac{1}{4}} - \frac{1}{18}z^{-9} - \frac{1}{9}\ln|z| + c}$

#7. (2 pts) First, do a couple of integrals to get the most general first derivative and function.

$$f'(x) = \int f''(x)\,dx = \int 12x^2 - 16x^{-3} + 18\,dx = 4x^3 + 8x^{-2} + 18x + c$$

$$f(x) = \int f(x)\,dx = \int 4x^3 + 8x^{-2} + 18x + c\,dx = x^4 - 8x^{-1} + 9x^2 + cx + d$$

Now plug in the condition to get a system of equations to be solved.

$$\begin{aligned} 8 &= f(1) = 2 + c + d \\ 77 &= f(-2) = 24 - 2x + d \end{aligned} \quad \Rightarrow \quad \begin{aligned} c &= -5 \\ d &= 11 \end{aligned}$$

The function is then,

$$\boxed{f(x) = x^4 - 8x^{-1} + 9x^2 - 5x + 11}$$

#9. (2 pts) $u = 4w + \mathbf{e}^{-6w}$ $\quad du = \left(4 - 6\mathbf{e}^{-6w}\right)dw = 2\left(2 - 3\mathbf{e}^{-6w}\right)dw \;\rightarrow\; \left(2 - 3\mathbf{e}^{-6w}\right)dw = \tfrac{1}{2}du$

$$\int \left(2 - 3\mathbf{e}^{-6w}\right)\sec^2\left(4w + \mathbf{e}^{-6w}\right)dw = \tfrac{1}{2}\int \sec^2(u)\,du = \tfrac{1}{2}\tan(u) + c = \boxed{\tfrac{1}{2}\tan\left(4w + \mathbf{e}^{-6w}\right) + c}$$

#11. (2 pts) This one can be a little tricky if you don't see how to do it.

$$u = \ln z \qquad du = \tfrac{1}{z}dz$$

$$\int \frac{6}{z\left[\ln(z)\right]^3}dz = \int \frac{6}{u^3}du = \int 6u^{-3}\,du = -3u^{-2} + c = \boxed{-3\left[\ln(z)\right]^{-2} + c}$$

Not Graded

#1. (a) $\int 3x^{-4} - 24x^7 + 7x - 8\,dx = \boxed{-x^{-3} - 3x^8 + \tfrac{7}{2}x^2 - 8x + c}$

(b) $\int 3x^{-4} - 24x^7\,dx + 7x - 8 = \boxed{-x^{-3} - 3x^8 + 7x - 8 + c}$

#4. $\int 4\mathbf{e}^x - 9\sin(x) + 3\sec(x)\tan(x)\,dx = \boxed{4\mathbf{e}^x + 9\cos(x) + 3\sec(x) + c}$

#5. $\int \left(t^2 - 3\right)(t + 4)\,dt = \int t^3 + 4t^2 - 3t - 12\,dt = \boxed{\tfrac{1}{4}t^4 + \tfrac{4}{3}t^3 - \tfrac{3}{2}t^2 - 12t + c}$

#6. $\displaystyle\int \frac{\cos(y)\sin(y)+7\csc(y)}{\sin(y)}\,dy = \int \cos(y)+7\csc^2(y)\,dy = \boxed{\sin(7)-7\cot(y)+c}$

#8. $u = t^2+\cos(t)$ $\quad du = (2t-\sin(t))\,dt = -(\sin(t)-2t)\,dt \quad \rightarrow \quad (\sin(t)-2t)\,dtdz = -du$

$$\int (\sin(t)-2t)\sqrt{t^2+\cos(t)}\,dt = -\int u^{\frac{1}{2}}\,du = -\tfrac{2}{3}u^{\frac{3}{2}}+c = \boxed{-\tfrac{2}{3}\left(t^2+\cos(t)\right)^{\frac{3}{2}}+c}$$

#10. $u = x^2-4x+1 \qquad du = (2x-4)\,dx$

$$\int \frac{2x-4}{x^2-4x+1}+\frac{-5(2x-4)}{\left(x^2-4x+1\right)^3}\,dx = \int \frac{1}{u}-\frac{5}{u^3}\,du = \int \frac{1}{u}-5u^{-3}\,du = \ln|u|+\tfrac{5}{2}u^{-2}+c$$

$$= \boxed{\ln\left|x^2-4x+1\right|+\tfrac{5}{2}\left(x^2-4x+1\right)^{-2}+c}$$

#12. In this case it's best to first break up the integral into two separate integrals.

$$\int \sqrt[5]{y^3}+\sec(1-y)\tan(1-y)\,dy = \int y^{\frac{3}{5}}\,dy+\int \sec(1-y)\tan(1-y)\,dy$$

Only the second integral needs the following substitution.

$$u = 1-y \qquad du = -dy \qquad dy = -du$$

The integral is then,

$$\int \sqrt[5]{y^3}+\sec(1-y)\tan(1-y)\,dy = \int y^{\frac{3}{5}}\,dy-\int \sec(u)\tan(u)\,du$$

$$= \tfrac{5}{8}y^{\frac{8}{5}}-\sec(u)+c = \boxed{\tfrac{5}{8}y^{\frac{8}{5}}-\sec(1-y)+c}$$