

#1. (2 pts) We'll need to break this up into two integrals and then do different substitutions on each.

$$\int \sin\left(\frac{x}{12}\right) - x^4 e^{2-x^5} dx = \int \sin\left(\frac{x}{12}\right) dx - \int x^4 e^{2-x^5} dx$$

Here are the substitutions for each.

$$u = \frac{x}{12} \quad du = \frac{1}{12} dx \quad dx = 12 du \quad || \quad v = 2 - x^5 \quad dv = -5x^4 dx \quad x^4 dx = -\frac{1}{5} dv$$

The integral is then,

$$\begin{aligned} \int \sin\left(\frac{x}{12}\right) - x^4 e^{2-x^5} dx &= 12 \int \sin(u) du + \frac{1}{5} \int e^v dv \\ &= -12 \cos(u) + \frac{1}{5} e^v + c = \boxed{-12 \cos\left(\frac{x}{12}\right) + \frac{1}{5} e^{2-x^5} + c} \end{aligned}$$

#4. (2 pts) This is one of those tricky substitutions in that you will need to use the substitution twice.

$$\begin{aligned} u &= 2 + x^5 & du &= 5x^4 dx & x^4 dx &= \frac{1}{5} du & x^5 &= u - 2 \\ \int x^9 (2 + x^5)^{-3} dx &= \int x^5 x^4 (2 + x^5)^{-3} dx = \frac{1}{5} \int (u - 2) u^{-3} du = \frac{1}{5} \int u^{-2} - 2u^{-3} du \\ &= \frac{1}{5} (-u^{-1} + u^{-2}) + c = \boxed{\frac{1}{5} \left((2 + x^5)^{-2} - (2 + x^5)^{-1} \right) + c} \end{aligned}$$

#8. (2 pts) To do this we just need to break up the integral and then use properties to get the first one to look like the formula given in class and then use the formula on both (with some chain rule of course).

$$\begin{aligned} \int_{x^3}^{4x} \cos(t^2) dt &= \int_{x^3}^0 \cos(t^2) dt + \int_0^{4x} \cos(t^2) dt \\ &= -\int_0^{x^3} \cos(t^2) dt + \int_0^{4x} \cos(t^2) dt \\ &= -3x^2 \cos\left((x^3)^2\right) + 4 \cos\left((4x)^2\right) = \boxed{-3x^2 \cos(x^6) + 4 \cos(16x^2)} \end{aligned}$$

#10. (2 pts)

$$\begin{aligned} \int_1^{-8} 9z^2 - 4z - 14z^{\frac{4}{3}} dz &= \left(3z^3 - 2z^2 - 6z^{\frac{7}{3}} \right) \Big|_1^{-8} \\ &= \left(-1536 - 128 - 6(-8)^{\frac{7}{3}} \right) - (3 - 2 - 6) \quad (-8)^{\frac{7}{3}} = -128 \\ &= -896 - (-5) = \boxed{-891} \end{aligned}$$

#13. (2 pts) In this case we can see that if $x < -2$ then $8 + 4x < 0$ and likewise if $x > -2$ then $8 + 4x > 0$ so,

$$\begin{aligned} \int_{-3}^0 |8 + 4x| dx &= \int_{-3}^{-2} |8 + 4x| dx + \int_{-2}^0 |8 + 4x| dx = \int_{-3}^{-2} -(8 + 4x) dx + \int_{-2}^0 8 + 4x dx \\ &= -(8x + 2x^2) \Big|_{-3}^{-2} + (8x + 2x^2) \Big|_{-2}^0 = (8 - (-6)) + (0 - (-8)) = \boxed{10} \end{aligned}$$

Not Graded

#2. We'll need to break this into two integrals and then do a separate substitution on each.

$$\int \frac{3+8t}{t^2+9} dt = \int \frac{3}{t^2+9} dt + \int \frac{8t}{t^2+9} dt = \frac{1}{9} \int \frac{3}{\frac{1}{9}t^2+1} dt + \int \frac{8t}{t^2+9} dt = \frac{3}{9} \int \frac{1}{\left(\frac{1}{3}t\right)^2+1} dt + \int \frac{8t}{t^2+9} dt$$

Here are the substitutions for each

$$u = \frac{1}{3}t \quad du = \frac{1}{3}dt \quad dt = 3du \quad || \quad v = t^2 + 9 \quad dv = 2t dt \quad tdt = \frac{1}{2}dv$$

The integral is then,

$$\int \frac{3+8t}{t^2+9} dt = \frac{1}{3}(3) \int \frac{1}{u^2+1} dt + \frac{8}{2} \int \frac{1}{v} dt = \tan^{-1}(u) + 4 \ln|v| + c = \boxed{\tan^{-1}\left(\frac{t}{3}\right) + 4 \ln|t^2+9| + c}$$

#3. $u = \frac{1}{z} \quad du = -\frac{1}{z^2} dz \quad \frac{1}{z^2} dz = -du$

$$\int \frac{\sec^2\left(\frac{1}{z}\right)}{z^2} dz = -\int \sec^2(u) du = -\tan(u) + c = \boxed{-\tan\left(\frac{1}{z}\right) + c}$$

#5. For this problem we have : $\Delta x = \frac{4-2}{2} = \frac{1}{2}$. So, the interval will be broken up into the following four subintervals.

$$[1, 1.5], [1.5, 2], [2, 2.5], [2.5, 3]$$

The midpoints of each of these are,

$$1.25, 1.75, 2.25, 2.75$$

The area is then approximately,

$$\text{Area} \approx \frac{1}{4} [g(1.25) + g(1.75) + g(2.25) + g(2.75)] = 701.6632$$

#6. This is really just a giant usage of the various definite integral properties.

$$\begin{aligned} \int_{-9}^{12} 2f(x) - 8g(x) dx &= 2 \int_{-9}^{12} f(x) dx - 8 \int_{-9}^{12} g(x) dx = 2 \int_{-9}^{12} f(x) dx + 8 \int_{12}^{-9} g(x) dx \\ &= 2(11) + 8(4) = \boxed{54} \end{aligned}$$

#7. We just need to break up the asked for integral and use various properties to get the various pieces broken up into the given integrals.

$$\begin{aligned} \int_1^{15} f(x) dx &= \int_1^{-6} f(x) dx + \int_{-6}^7 f(x) dx + \int_7^{15} f(x) dx \\ &= \int_1^{-6} f(x) dx - \int_7^{-6} f(x) dx - \int_{15}^7 f(x) dx = -13 - (-3) - (10) = \boxed{-20} \end{aligned}$$

#9.

$$\begin{aligned}
 \int_0^{\frac{\pi}{4}} 2 \cos \theta - 8 \sin \theta + 4 d\theta &= (2 \sin \theta + 8 \cos \theta + 4\theta) \Big|_0^{\frac{\pi}{4}} \\
 &= \left(2 \sin \left(\frac{\pi}{4} \right) + 8 \cos \left(\frac{\pi}{4} \right) + 4 \left(\frac{\pi}{4} \right) \right) - (2 \sin(0) + 8 \cos(0) + 4(0)) \\
 &= \boxed{5\sqrt{2} + \pi - 8 = 2.21266}
 \end{aligned}$$

#11.

$$\begin{aligned}
 \int_2^4 \frac{1}{2x^4} + \frac{4}{x} - 8e^x dx &= \left(-\frac{1}{6x^3} + 4 \ln|x| - 8e^x \right) \Big|_2^4 \\
 &= \left(-\frac{1}{384} + 4 \ln|4| - 8e^4 \right) - \left(-\frac{1}{48} + 4 \ln|2| - 8e^2 \right) \\
 &= \boxed{\frac{7}{384} + 4(\ln(4) - \ln(2)) - 8(e^4 - e^2) = -374.8819}
 \end{aligned}$$

#12. We'll need to break this up at $t = 2$ so we can plug in the appropriate formula.

$$\begin{aligned}
 \int_1^5 f(t) dt &= \int_1^3 f(t) dt + \int_3^5 f(t) dt = \int_1^3 12t^3 dt + \int_3^5 1 - 4t dt \\
 &= 3t^4 \Big|_1^3 + (t - 2t^2) \Big|_3^5 = (243 - 3) + (-45 - (-15)) = \boxed{210}
 \end{aligned}$$

#14. It is not possible to do this integral because we have division by zero at $x = 0$ in the third term and $x = 0$ is in the interval of integration. Note that the fact that we can integrate the first two terms is immaterial here. There is a problem with the third term and so the whole integral can't be done.