Math 2413 **Homework Set 11** **10 Points**

#1. (2 pts)

$u = \frac{\pi}{2}t \quad du = \frac{\pi}{2}dt \qquad dt = \frac{2}{\pi}du \qquad t = 1 \to u = \frac{\pi}{2},\ t = 3 \to u = \frac{3\pi}{2}$

$$\int_1^3 \cos\left(\frac{\pi t}{2}\right)dt = \frac{2}{\pi}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos(u)\,du = \frac{2}{\pi}\left(\sin(u)\right)\Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = \frac{2}{\pi}\left(\sin\left(\tfrac{3\pi}{2}\right) - \sin\left(\tfrac{\pi}{2}\right)\right)\boxed{= -\tfrac{4}{\pi}}$$

#3. (2 pts) We'll need to split this up and do two substitutions.

$$\int_0^1 \sqrt{2+8x} + \frac{6x}{x^2+1}\,dx = \int_0^1 \sqrt{2+8x}\,dx + \int_0^1 \frac{6x}{x^2+1}\,dx$$

$u = 2+8x \qquad du = 8dx \qquad dx = \frac{1}{8}du \qquad x = 0 \to u = 2,\ x = 1 \to u = 10$

$v = x^2+1 \qquad dv = 2xdx \qquad xdx = \frac{1}{2}dv \qquad x = 0 \to u = 1,\ x = 1 \to u = 2$

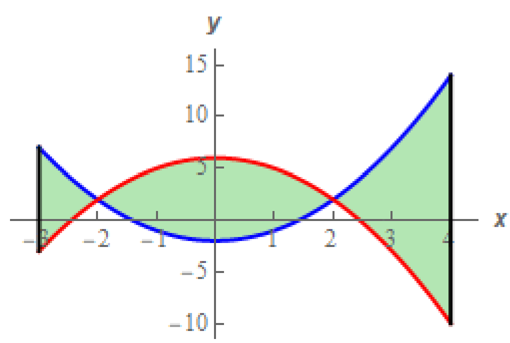
$$\int_0^1 \sqrt{2+8x} + \frac{6x}{x^2+1}\,dx = \frac{1}{8}\int_2^{10} u^{\frac{1}{2}}\,du + 3\int_1^2 \frac{1}{v}\,dx = \frac{1}{8}\left(\tfrac{2}{3}\right)u^{\frac{3}{2}}\Big|_2^{10} + 3\ln|v|\Big|_1^2$$

$$= \frac{1}{12}\left(10^{\frac{3}{2}} - 2^{\frac{3}{2}}\right) + 3\left(\ln 2 - \ln 1\right) = \boxed{\frac{1}{12}\left(10^{\frac{3}{2}} - 2^{\frac{3}{2}}\right) + 3\ln 2 = 4.4790}$$

#7. (2 pts) A sketch is to the right. Here are the intersection points.

$$x^2 - 2 = 6 - x^2$$

$$2x^2 = 8 \quad \Rightarrow \quad \underline{x = -2, 2}$$



The area is then going to compute need three integrals to get the area.

$$A = \int_{-3}^{-2} x^2 - 2 - \left(6 - x^2\right)dx + \int_{-2}^{2} 6 - x^2 - \left(x^2 - 2\right)dx + \int_2^4 x^2 - 2 - \left(6 - x^2\right)dx$$

$$= \int_{-3}^{-2} 2x^2 - 8\,dx + \int_{-2}^{2} 8 - 2x^2\,dx + \int_2^4 2x^2 - 8\,dx$$

$$= \left(\tfrac{2}{3}x^3 - 8x\right)\Big|_{-3}^{-2} + \left(8x - \tfrac{2}{3}x^3\right)\Big|_{-2}^{2} + \left(\tfrac{2}{3}x^3 - 8x\right)\Big|_2^4 = \frac{14}{3} + \frac{64}{3} + \frac{64}{3} = \boxed{\frac{142}{3} = 47.3333}$$

#10. **(2 pts)** A sketch is to the right. The radii are,

$$\text{o.r.} = 4 - \left(y^2 - 1\right) = 5 - y^2 \qquad \text{i.r.} = 4 - \left(1 - y\right) = y + 3$$

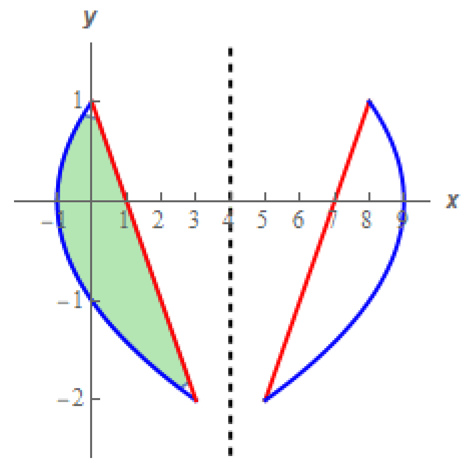
Here is the cross sectional area.

$$A = \pi\left[\left(5 - y^2\right)^2 - \left(y + 3\right)^2\right] = \pi\left(16 - 6y - 11y^2 + y^4\right)$$

The volume is then,

$$V = \pi\int_{-2}^{1} 16 - 6y - 11y^2 - y^4\,dy$$

$$= \pi\left(16y - 3y^2 - \tfrac{11}{3}y^3 - \tfrac{1}{5}y^5\right)\Big|_{-2}^{1} = \boxed{\tfrac{153}{5}\pi}$$

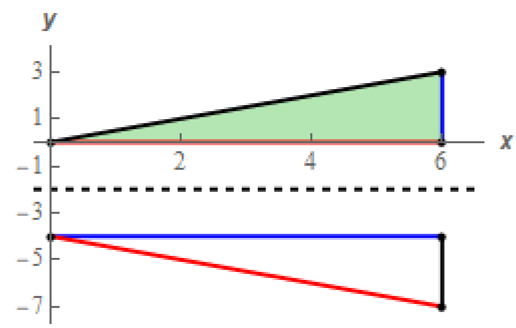


#13. **(2 pts)** A sketch is to the right. In this case we'll need the equation of the hypotenuse written as : $x = 2y$. The radius and height are,

$$\text{radius} = y + 2 \qquad \text{width} = 6 - 2y$$

The cross sectional area is then,

$$A = 2\pi\left(y + 2\right)\left(6 - 2y\right) = 2\pi\left(12 + 2y - 2y^2\right)$$



The volume is then,

$$V = 2\pi\int_{0}^{3} 12 + 2y - 2y^2\,dy = 2\pi\left(12y + y^2 - \tfrac{2}{3}y^3\right)\Big|_{0}^{3} = \boxed{54\pi}$$

Not Graded

#2. We'll need to split this up and then do a substitution on the second integral.

$$\int_{-1}^{3} 6y - 8y\left(2 - y^2\right)^3\,dy = \int_{-1}^{3} 6y\,dy - \int_{-1}^{3} 8y\left(2 - y^2\right)^3\,dy$$

The substitution for the second integral is,

$$u = 2 - y^2 \qquad du = -2y\,dt \qquad y\,dt = -\tfrac{1}{2}du \qquad y = -1 \rightarrow u = 1,\; t = 3 \rightarrow u = -7$$

$$\int_{-1}^{3} 6y - 8y\left(2 - y^2\right)^3\,dy = \int_{-1}^{3} 6y\,dy + 4\int_{1}^{-7} u^3\,dy = 3y^2\Big|_{-1}^{3} + u^4\Big|_{1}^{-7}$$

$$= \left(27 - 3\right) + \left(2401 - 1\right) = \boxed{2424}$$

#4.

$$\int_{-1}^{1} \mathbf{e}^{-2x} - \frac{4}{3x + 6}\,dx = \int_{-1}^{1} \mathbf{e}^{-2x}\,dx - \int_{-1}^{1} \frac{4}{3x + 6}\,dx$$

Here are the substitutions.

$$u = -2x \qquad du = -2dx \qquad dx = -\tfrac{1}{2}du \qquad x = -1 \to u = 2,\; x = 1 \to u = -2$$
$$v = 3x + 6 \qquad dv = 3dx \qquad dx = \tfrac{1}{3}dv \qquad x = -1 \to u = 3,\; x = 1 \to u = 9$$

The integral is then,

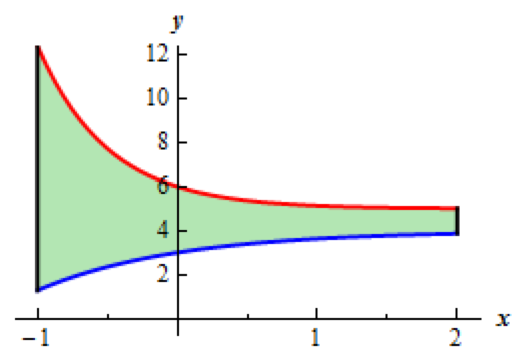
$$\int_{-1}^{1} \mathbf{e}^{-2x} - \frac{4}{3x+6}\,dx = -\frac{1}{2}\int_{2}^{-2} \mathbf{e}^{u}\,du + \frac{4}{3}\int_{3}^{9} \frac{1}{v}\,dx = -\frac{1}{2}\mathbf{e}^{u}\Big|_{2}^{-2} + \frac{4}{3}5\ln|v|\Big|_{3}^{9}$$
$$= -\tfrac{1}{2}\left(\mathbf{e}^{-2} - \mathbf{e}^{2}\right) + \tfrac{4}{3}\left(\ln|9| - \ln|3|\right) = \boxed{2.1620}$$

#5. Unlike **#4** this integral can't be done. We have a division by zero issue at $x = -2$ (in the second term) and that is in the interval for this integral while it wasn't in the interval for **#4** and so we could do that integral.

#6. A sketch is to the right. Here's the area.

$$A = \int_{-1}^{2} 5 + \mathbf{e}^{-2x} - \left(4 - \mathbf{e}^{-x}\right)dx$$
$$= \int_{-1}^{2} 1 + \mathbf{e}^{-2x} + \mathbf{e}^{-x}\,dx = \left(x - \tfrac{1}{2}\mathbf{e}^{-2x} - \mathbf{e}^{-x}\right)\Big|_{-1}^{2}$$
$$= \boxed{3 - \tfrac{1}{2}\mathbf{e}^{-4} - \mathbf{e}^{-2} + \tfrac{1}{2}\mathbf{e}^{2} + \mathbf{e} = 9.2683}$$

For the 2nd term I used the substitution $u = -2x$ and for the 3rd term I used $v = -x$ for the substitution.

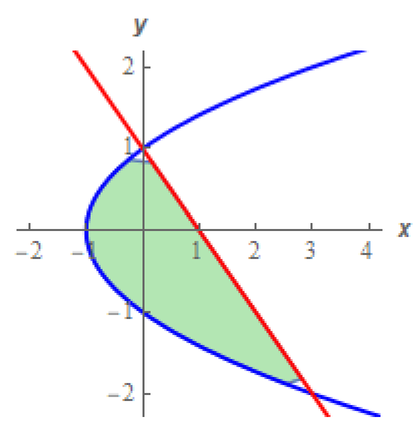


#8. A sketch is to the right. Here are the intersection points.

$$y^2 - 1 = 1 - y$$
$$y^2 + y - 2 = (y+2)(y-1) = 0 \quad \Rightarrow \quad \underline{y = -2, 1}$$

The area is then,

$$A = \int_{-2}^{1} 1 - y - \left(y^2 - 1\right)dy = \int_{-2}^{1} 2 - y - y^2\,dy$$
$$= \left(2y - \tfrac{1}{2}y - \tfrac{1}{3}y^3\right)\Big|_{-2}^{1} = \boxed{\tfrac{9}{2}}$$



#9. A sketch is to the right. The radii are,

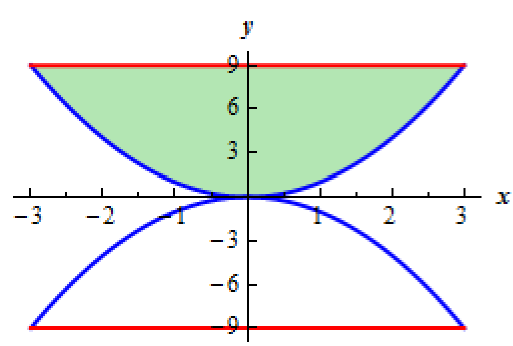
$$\text{o.r.} = 9 \qquad \text{i.r.} = x^2$$

Here is the cross sectional area.

$$A = \pi\left[(9)^2 - \left(x^2\right)^2\right] = \pi\left(81 - x^4\right)$$

The volume is then,

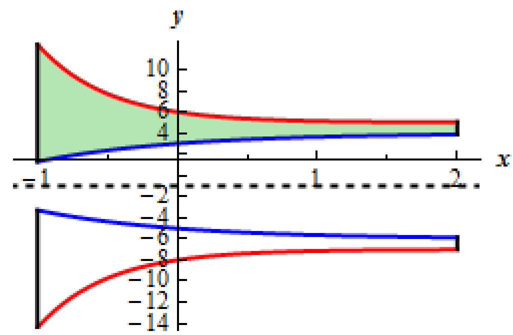
$$V = \pi\int_{-3}^{3} 81 - x^4\,dx = \pi\left(81x - \tfrac{1}{5}x^5\right)\Big|_{-3}^{3} = \boxed{\tfrac{1944}{5}\pi}$$



<u>#11.</u> A sketch is to the right. The radii are,

$$\text{o.r.} = 1 + \left(5 + \mathbf{e}^{-2x}\right) = 6 + \mathbf{e}^{-2x}$$

$$\text{i.r.} = 1 + \left(4 - \mathbf{e}^{-x}\right) = 5 - \mathbf{e}^{-x}$$



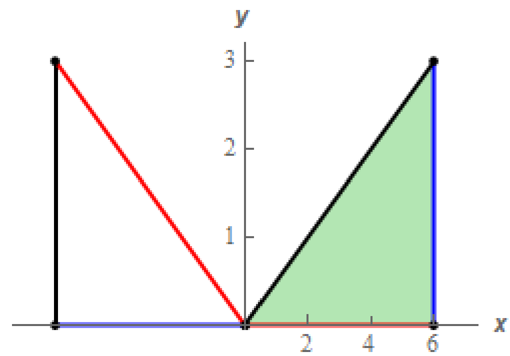
Here is the cross sectional area.

$$A = \pi\left[\left(6 + \mathbf{e}^{-2x}\right)^2 - \left(5 - \mathbf{e}^{-x}\right)^2\right]$$

The integral giving the volume is then,

$$\boxed{V = \pi\int_{-1}^{2}\left(6 + \mathbf{e}^{-2x}\right)^2 - \left(5 - \mathbf{e}^{-x}\right)^2}$$

<u>#12.</u> A sketch is to the right. Since we know two points on the hypotenuse of the triangle we can easily find the equation of the hypotenuse : $y = \frac{1}{2}x$. The radius and height for the cylinder are,



$$\text{radius} = x \qquad \text{height} = \tfrac{1}{2}x$$

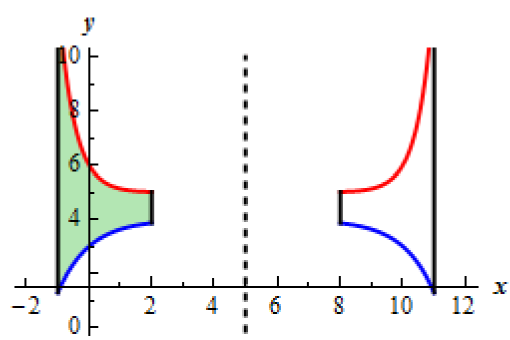
The cross sectional area is then,

$$A = 2\pi x\left(\tfrac{1}{2}x\right) = \pi x^2$$

The volume is then,

$$V = \pi\int_0^6 x^2\,dx = \pi\left(\tfrac{1}{3}x^3\right)\Big|_0^6 = \boxed{72\pi}$$

<u>#14.</u> A sketch is to the right. The radius and height are,



$$\text{radius} = 5 - x$$

$$\text{height} = 5 + \mathbf{e}^{-2x} - \left(4 - \mathbf{e}^{-x}\right) = 1 + \mathbf{e}^{-2x}\mathbf{e}^{-x}$$

The cross sectional area is then,

$$A = 2\pi\left(5 - x\right)\left(1 + \mathbf{e}^{-2x}\mathbf{e}^{-x}\right)$$

The volume is then,

$$\boxed{V = 2\pi\int_{-1}^{2}\left(5 - x\right)\left(1 + \mathbf{e}^{-2x}\mathbf{e}^{-x}\right)dx}$$