4. (2 pts) First do a rewrite with a trig formula as suggested and then split up the integral.

$$\int_0^{\frac{\pi}{2}} 6t\sin(2t)\cos(2t)\,dt = \int_0^{\frac{\pi}{2}} 3t\sin(4t)\,dt$$

For the second integral we have : $u = 3t \qquad du = 3dy \qquad dv = \sin(4t)\,dt \quad v = -\frac{1}{4}\cos(4t)$

$$\int_0^{\frac{\pi}{2}} 6t\sin(2t)\cos(2t)\,dt = \left[-\tfrac{3}{4}t\cos(4t) + \tfrac{3}{4}\int\cos(4t)\right]\Bigg|_0^{\frac{\pi}{2}}$$

$$= \left[-\tfrac{3}{4}t\cos(4t) + \tfrac{3}{16}\sin(4t)\right]\Bigg|_0^{\frac{\pi}{2}} = \boxed{-\tfrac{3\pi}{8}}$$

5. (2 pts) $u = y^6 \qquad du = 6y^5dt \qquad dv = y^5\cos\left(1-2y^6\right)dy \qquad v = -\frac{1}{12}\sin\left(1-2y^6\right)$

$$\int y^{11}\cos\left(1-2y^6\right)dy = -\tfrac{1}{12}y^6\sin\left(1-2y^6\right) + \tfrac{1}{2}\int y^5\sin\left(1-2y^6\right)dy$$

$$= \boxed{-\tfrac{1}{12}y^6\sin\left(1-2y^6\right) + \tfrac{1}{24}\cos\left(1-2y^6\right) + c}$$

Note that as an alternate solution method the Cacl I substitution $u = 1-2t^6$ would reduce the integral to $-\frac{1}{12}\int\frac{1}{2}(1-u)\sin(u)\,du$ which you could then do integration by parts to evaluate.

6. (2 pts)

$$\int 3y\cos^5\left(y^2\right)\sin^6\left(y^2\right)dy = \int 3y\left(1-\sin^2\left(y^2\right)\right)^2\sin^6\left(y^2\right)\cos\left(y^2\right)dy \qquad\qquad u = \sin\left(y^2\right)$$

$$= \tfrac{3}{2}\int\left(1-u^2\right)^2u^6\,du = \tfrac{3}{2}\int u^6 - 2u^8 + u^{10}\,du$$

$$= \boxed{\tfrac{3}{2}\left(\tfrac{1}{7}\sin^7\left(y^2\right) - \tfrac{2}{9}\sin^9\left(y^2\right) + \tfrac{1}{11}\sin^{11}\left(y^2\right)\right) + c}$$

9. (2 pts)

$$\int\frac{3\sin(x)+\sec^4(x)}{\tan(x)}dx = \int\frac{3\sin(x)}{\tan(x)} + \frac{\sec^4(x)}{\tan(x)}dx = \int 3\cos(x) + \frac{\sec^4(x)}{\tan(x)}dx$$

$$= \int 3\cos(x)\,dx + \int\frac{\tan^2(x)+1}{\tan(x)}\sec^2(x)\,dx$$

$$= \int 3\cos(x)\,dx + \int\frac{u^2+1}{u}du = \int 3\cos(x)\,dx + \int u + \frac{1}{u}du$$

$$= 3\sin(x) + \tfrac{1}{2}\tan^2(x) + \ln\left|\tan(x)\right| + c$$

10. (2 pts)

$$\int \cot^3\left(\tfrac{1}{4}t\right)\csc^5\left(\tfrac{1}{4}t\right)dt = \int \cot^2\left(\tfrac{1}{4}t\right)\csc^4\left(\tfrac{1}{4}t\right)\cot\left(\tfrac{1}{4}t\right)\csc\left(\tfrac{1}{4}t\right)dt$$

$$= \int \left(\csc^2\left(\tfrac{1}{4}t\right)-1\right)\csc^4\left(\tfrac{1}{4}t\right)\cot\left(\tfrac{1}{4}t\right)\csc\left(\tfrac{1}{4}t\right)dt \qquad u=\csc\left(\tfrac{1}{4}t\right)$$

$$= -4\int\left(u^2-1\right)u^4du = -4\int u^6-u^4du$$

$$= \boxed{-4\left(\tfrac{1}{7}\csc^7\left(\tfrac{1}{4}t\right)-\tfrac{1}{5}\csc^5\left(\tfrac{1}{4}t\right)\right)+c}$$

Not Graded

1. $u=8x-x^2 \qquad du=(8-2x)dx \qquad dv=\mathbf{e}^{-2x}dx \qquad v=-\tfrac{1}{2}\mathbf{e}^{-2x}$

$$\int_0^{-2}\left(8x-x^2\right)\mathbf{e}^{-2x}\,dx = \left[-\tfrac{1}{2}\mathbf{e}^{-2x}\left(8x-x^2\right)+\tfrac{1}{2}\int(8-2x)\mathbf{e}^{-2x}\,dx\right]\Bigg|_0^{-2}$$

$$u=8-2x \qquad du=-2dx \qquad dv=\mathbf{e}^{-2x}dx \qquad v=-\tfrac{1}{2}\mathbf{e}^{-2x}$$

$$\int_0^{-2}\left(8x-x^2\right)\mathbf{e}^{-2x}\,dx = \left[-\tfrac{1}{2}\mathbf{e}^{-2x}\left(8x-x^2\right)+\tfrac{1}{2}\left(-\tfrac{1}{2}\mathbf{e}^{-2x}(8-2x)+\int\mathbf{e}^{-2x}\,dx\right)\right]\Bigg|_0^{-2}$$

$$= \left[-\tfrac{1}{2}\mathbf{e}^{-2x}\left(8x-x^2\right)-\tfrac{1}{4}\mathbf{e}^{-2x}(8-2x)+\tfrac{1}{4}\mathbf{e}^{-2x}\,dx\right]\Bigg|_0^{-2} = \boxed{\tfrac{29}{4}\mathbf{e}^4+\tfrac{7}{4}=397.587}$$

2. $u=\cos\left(\tfrac{t}{3}\right) \qquad du=-\tfrac{1}{3}\sin\left(\tfrac{t}{3}\right)dt \qquad dv=\mathbf{e}^{-t}dt \qquad v=-\mathbf{e}^{-t}$

$$\int\mathbf{e}^{-t}\cos\left(\tfrac{t}{3}\right)dt = -\mathbf{e}^{-t}\cos\left(\tfrac{t}{3}\right)-\tfrac{1}{3}\int\mathbf{e}^{-t}\sin\left(\tfrac{t}{3}\right)dt$$

$$u=\sin\left(\tfrac{t}{3}\right) \qquad du=\tfrac{1}{3}\cos\left(\tfrac{t}{3}\right)dt \qquad dv=\mathbf{e}^{-t}dt \qquad v=-\mathbf{e}^{-t}$$

$$\int\mathbf{e}^{-t}\cos\left(\tfrac{t}{3}\right)dt = -\mathbf{e}^{-t}\cos\left(\tfrac{t}{3}\right)-\tfrac{1}{3}\left(-\mathbf{e}^{-t}\sin\left(\tfrac{t}{3}\right)+\tfrac{1}{3}\int\mathbf{e}^{-t}\cos\left(\tfrac{t}{3}\right)dt\right)$$

$$= -\mathbf{e}^{-t}\cos\left(\tfrac{t}{3}\right)+\tfrac{1}{3}\mathbf{e}^{-t}\sin\left(\tfrac{t}{3}\right)-\tfrac{1}{9}\int\mathbf{e}^{-t}\cos\left(\tfrac{t}{3}\right)dt$$

$$\tfrac{10}{9}\int\mathbf{e}^{-t}\cos\left(\tfrac{t}{3}\right)dt = -\mathbf{e}^{-t}\cos\left(\tfrac{t}{3}\right)+\tfrac{1}{3}\mathbf{e}^{-t}\sin\left(\tfrac{t}{3}\right)$$

$$\int\mathbf{e}^{-t}\cos\left(\tfrac{t}{3}\right)dt = \boxed{\tfrac{9}{10}\left(-\mathbf{e}^{-t}\cos\left(\tfrac{t}{3}\right)+\tfrac{1}{3}\mathbf{e}^{-t}\sin\left(\tfrac{t}{3}\right)\right)+c}$$

3. $u=3\sin^{-1}(4z) \qquad du=\dfrac{12}{\sqrt{1-(4z)^2}}dz=\dfrac{12}{\sqrt{1-16z^2}}dz \qquad dv=dz \qquad v=z$

$$\int 3\sin^{-1}(4z)\,dz = 3z\sin^{-1}(4z)-\int\frac{12z}{\sqrt{1-16z^2}}dz = \boxed{3z\sin^{-1}(4z)+\tfrac{3}{4}\sqrt{1-16z^2}+c}$$

7.

$$\int \sin^4(2x)\,dx = \int\left(\sin^2(2x)\right)^2 dx = \int\left(\tfrac{1}{2}\left(1-\cos(4x)\right)\right)^2 dx = \tfrac{1}{4}\int 1-2\cos(4x)+\cos^2(4x)\,dx$$

$$= \tfrac{1}{4}\int 1-2\cos(4x)+\tfrac{1}{2}\left(1+\cos(8x)\right)dx + \tfrac{1}{4}\int \tfrac{3}{2}-2\cos(4x)+\tfrac{1}{2}\cos(8x)\,dx$$

$$= \boxed{\tfrac{1}{4}\left(\tfrac{3}{2}x-\tfrac{1}{2}\sin(4x)+\tfrac{1}{16}\sin(8x)\right)+c}$$

8.

$$\int \tan^2(8z)\sec^8(8z)\,dz = \int \tan^2(8z)\left(\sec^2(8z)\right)^3\sec^2(8z)\,dz$$

$$= \int \tan^2(8z)\left(\tan^2(8z)+1\right)^3\sec^2(8z)\,dz\,dz$$

$$= \int \tan^2(8z)\left[\tan^6(8z)+3\tan^4(8z)+3\tan^2(8z)+1\right]\sec^2(8z)\,dz$$

$$= \tfrac{1}{8}\int u^2\left(u^6+3u^4+3u^2+1\right)du \qquad u=\tan(8z)$$

$$= \tfrac{1}{8}\int u^9+3u^6+3u^4+u^2\,du$$

$$= \boxed{\tfrac{1}{8}\left(\tfrac{1}{9}\tan^9(8z)+\tfrac{3}{7}\tan^7(8z)+\tfrac{3}{5}\tan^5(8z)+\tfrac{1}{3}\tan^3(8z)+c\right)}$$