

3. (2 pts) A quick substitution ($u = \ln(2x)$) will put this into a form that we can then use partial fractions on.

$$\int \frac{[\ln(2x)]^2}{x([\ln(2x)]^2 + 2\ln(2x) - 15)} dx = \int \frac{u^2}{u^2 + 2u - 15} du = \int 1 + \frac{15 - 2u}{(u-3)(u+5)} du$$

$$\frac{2u - 15}{(u-3)(u+5)} = \frac{A}{u-3} + \frac{B}{u+5} \quad \Rightarrow \quad 15 - 2u = A(u+5) + B(u-3)$$

$$u = 3: \quad 9 = 8A$$

$$A = \frac{9}{8}$$

$$u = -5: \quad 25 = -8B$$

$$B = -\frac{25}{8}$$

$$\int \frac{[\ln x]^2}{x([\ln x]^2 - 4\ln x - 12)} dx = \int 1 + \frac{\frac{9}{8}}{u-3} - \frac{\frac{25}{8}}{u+5} du = u + \frac{9}{8} \ln|u-3| - \frac{25}{8} \ln|u+5| + c$$

$$= \boxed{\ln(2x) + \frac{9}{8} \ln|\ln(2x) - 3| - \frac{25}{8} \ln|\ln(2x) + 5| + c}$$

4. (3 pts) We'll use the following trig substitution.

$$y^2 = \frac{5}{2} \tan \theta \quad 2y dy = \frac{5}{2} \sec^2 \theta d\theta \quad \sqrt{25 + 4y^4} = \sqrt{25 + 25 \tan^2 \theta} = 5|\sec \theta| = 5 \sec \theta$$

$$\begin{aligned} \int y^7 \sqrt{25 + 4y^4} d\theta &= \int \left(\frac{5}{2} \tan \theta\right)^3 (5 \sec \theta) \left(\frac{5}{4} \sec^2 \theta\right) d\theta = \frac{3125}{32} \int \tan^3 \theta \sec^3 \theta d\theta \\ &= \frac{3125}{32} \int (\sec^2 \theta - 1) \sec^2 \theta \tan \theta \sec \theta d\theta = \frac{3125}{32} \int (u^2 - 1) u^2 du \\ &= \frac{3125}{32} \int u^4 - u^2 du = \frac{3125}{32} \left(\frac{1}{5} \sec^5 \theta - \frac{1}{3} \sec^3 \theta\right) + c \end{aligned}$$

From a quick right triangle we get that $\sec \theta = \frac{\sqrt{25+4y^4}}{5}$. The integral is then,

$$\int y^7 \sqrt{25 + 4y^4} d\theta = \boxed{\frac{1}{160} (25 + 4y^4)^{\frac{5}{2}} - \frac{25}{96} (25 + 4y^4)^{\frac{3}{2}} + c}$$

6. (2 pts) First, let's do the improper integral since we'll need that eventually.

$$\int x e^{x^2+1} dx = \frac{1}{2} e^{x^2+1}$$

Now, for the actual integral. We'll need to split it up and $x = 0$ seems like as good a point as any.

$$\begin{aligned} \int_{-\infty}^{\infty} x e^{x^2+1} dx &= \int_{-\infty}^0 x e^{x^2+1} dx + \int_0^{\infty} x e^{x^2+1} dx \\ &= \lim_{t \rightarrow -\infty} \int_t^0 x e^{x^2+1} dx + \lim_{s \rightarrow \infty} \int_0^s x e^{x^2+1} dx \\ &= \lim_{t \rightarrow -\infty} \left(\frac{1}{2} e^{x^2+1}\right) \Big|_t^0 + \lim_{s \rightarrow \infty} \left(\frac{1}{2} e^{x^2+1}\right) \Big|_0^s \\ &= \lim_{t \rightarrow -\infty} \left(\frac{1}{2} e - \frac{1}{2} e^{t^2+1}\right) + \lim_{s \rightarrow \infty} \left(\frac{1}{2} e^{s^2+1} - \frac{1}{2} e\right) \\ &= \quad \quad \quad -\infty \quad \quad + \quad \quad \quad \infty \end{aligned}$$

So, the both of the integrals are divergent and so the original integral is also **divergent**.

7. (3 pts) First do the improper integral since we'll need that eventually.

$$\int \frac{x+1}{x^2-4} dx = \int \frac{x+1}{(x-2)(x+2)} dx = \int \frac{\frac{3}{4}}{x-2} + \frac{\frac{1}{4}}{x+2} dx = \frac{3}{4} \ln|x-2| + \frac{1}{4} \ln|x+2|$$

$$\frac{x+1}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2} \Rightarrow x+1 = A(x+2) + B(x-2)$$

$$x=2: \quad 3 = 4A \quad \Rightarrow \quad A = \frac{3}{4}$$

$$x=-2: \quad -1 = -4B \quad \Rightarrow \quad B = \frac{1}{4}$$

From this work we can see that we'll need to split the integral up at $x = 1$ because we'll have division by zero at that point.

$$\begin{aligned} \int_0^3 \frac{x+1}{x^2-4} dx &= \int_0^2 \frac{x+1}{x^2-4} dx + \int_2^3 \frac{x+1}{x^2-4} dx \\ &= \lim_{t \rightarrow 2^-} \int_0^t \frac{x+1}{x^2-4} dx + \lim_{s \rightarrow 2^+} \int_s^3 \frac{x+1}{x^2-4} dx \\ &= \lim_{t \rightarrow 2^-} \left(\frac{3}{4} \ln|x-2| + \frac{1}{4} \ln|x+2| \right) \Big|_0^t + \lim_{s \rightarrow 2^+} \left(\frac{3}{4} \ln|x-2| + \frac{1}{4} \ln|x+2| \right) \Big|_s^3 \\ &= \lim_{t \rightarrow 2^-} \left(\frac{3}{4} \ln|t-2| + \frac{1}{4} \ln|t+2| - (\ln(2)) \right) + \lim_{s \rightarrow 2^+} \left(\frac{1}{4} \ln(5) - \left(\frac{3}{4} \ln|s-2| + \frac{1}{4} \ln|s+2| \right) \right) \\ &= \qquad \qquad \qquad -\infty \qquad \qquad \qquad + \qquad \qquad \qquad (-\infty) \end{aligned}$$

Both integrals are divergent because $\lim_{x \rightarrow 2} \ln|x-2| = -\infty$ and so the original integral is also **divergent**.

Not Graded

1. Make sure to use the double angle formula on the sine to get the arguments the same.

$$\begin{aligned} \int \cos^4(4t) \sin^3(8t) dt &= \int \cos^4(4t) (2 \sin(4t) \cos(4t))^3 dt \\ &= 8 \int \cos^7(4t) \sin^3(4t) dt = 8 \int \cos^7(4t) (1 - \cos^2(4t)) \sin(4t) dt \\ &= -2 \int u^7 (1 - u^2) du = -2 \int u^7 - u^9 du = \boxed{\frac{1}{5} \cos^{10}(4t) - \frac{1}{4} \cos^8(4t) + c} \end{aligned}$$

2. You'll need to square this out and then do some integration by parts on the last integral.

$$\begin{aligned} \int (2z + e^{4z})^2 dz &= \int 4z^2 + 4ze^{4z} + e^{8z} dz = \int 4z^2 + e^{8z} dz + \int 4ze^{4z} dz \\ u &= 4z \quad du = 4dz \quad dv = e^{4z} dz \quad v = \frac{1}{4} e^{4z} \\ \int (2z + e^{4z})^2 dz &= \int 4z^2 + e^{8z} dz + \left(ze^{4z} - \int e^{4z} dz \right) = \boxed{\frac{4}{3} z^3 + \frac{1}{8} e^{8z} + ze^{4z} - \frac{1}{4} e^{4z} + c} \end{aligned}$$

5.

$$u = t^8 \quad du = 8t^7 dt \quad dv = t^7 e^{1-t^8} dt \quad v = -\frac{1}{8} e^{1-t^8}$$

$$\int t^{15} e^{1-t^8} dt = -\frac{1}{8} t^8 e^{1-t^8} + \int t^7 e^{1-t^8} dt = \boxed{-\frac{1}{8} t^8 e^{1-t^8} - \frac{1}{8} e^{1-t^8} + c}$$

8. First do the improper integral since we'll need that eventually.

$$u = \ln(2x) \quad du = \frac{1}{x} dx \quad dv = x^2 dx \quad v = \frac{1}{3} x^3$$

$$\int x^2 \ln(2x) dx = \frac{1}{3} x^3 \ln(2x) - \frac{1}{3} \int x^2 dx = \frac{1}{3} x^3 \ln(2x) - \frac{1}{9} x^3$$

In this case we have an infinite limit as well as continuity problems at zero and so this is a combination of the two types of improper integrals that we looked at in lecture. To deal with this we'll need to split the integral up to get each problem into its own integral which we can then deal with. I'll use $x = \frac{1}{2}$ as the cutoff point as this will evaluate the logarithm to zero....

$$\begin{aligned} \int_0^{\infty} x^2 \ln(2x) dx &= \int_0^{\frac{1}{2}} x^2 \ln(2x) dx + \int_{\frac{1}{2}}^{\infty} x^2 \ln(2x) dx \\ &= \lim_{t \rightarrow 0^+} \int_t^{\frac{1}{2}} x^2 \ln(2x) dx + \lim_{s \rightarrow \infty} \int_{\frac{1}{2}}^s x^2 \ln(2x) dx \\ &= \lim_{t \rightarrow 0^+} \left(\frac{1}{3} x^3 \ln(2x) - \frac{1}{9} x^3 \right) \Big|_t^{\frac{1}{2}} + \lim_{s \rightarrow \infty} \left(\frac{1}{3} x^3 \ln(2x) - \frac{1}{9} x^3 \right) \Big|_{\frac{1}{2}}^s \\ &= \lim_{t \rightarrow 0^+} \left(-\frac{1}{72} - \frac{1}{3} t^3 \ln(2t) + \frac{1}{9} t^3 \right) + \lim_{s \rightarrow \infty} \left(\frac{1}{3} s^3 \ln(2s) - \frac{1}{9} s^3 + \frac{1}{72} \right) \\ &= \lim_{t \rightarrow 0^+} \left(-\frac{1}{72} - \frac{1}{3} t^3 \ln(2t) + \frac{1}{9} t^3 \right) + \lim_{s \rightarrow \infty} \left(\frac{1}{9} s^3 (3 \ln(2s) - 1) + \frac{1}{72} \right) \\ &= -\frac{1}{27} \qquad \qquad \qquad + \infty \end{aligned}$$

The first integral is convergent while the second is divergent so the original integral is **divergent**. Note that the first limit required a little rewrite and then L'Hospital's Rule. You can do that correct?

9. This looks like it should converge so we'll need a larger function that we know will converge. Also, we'll need to take advantage of the fact that e^{-x} is a decreasing function and so for $x \geq 1$ (i.e. our interval of integration) we will have $e^{-x} \leq e^{-1} < e^0 = 1$.

$$\int_1^{\infty} \frac{x e^{-x} - \sin^2(x)}{x^4} dx \leq \frac{x e^{-x}}{x^4} \qquad \sin^2(x) \geq 0 \quad \& \quad x e^{-x} - \sin^2(x) \leq x e^{-x}$$

$$\leq \frac{x}{x^4} = \frac{1}{x^3} \qquad e^{-x} \leq 1$$

Now, we know that $\int_1^{\infty} \frac{1}{x^3} dx$ will converge because $p = 3 > 1$ and so by the comparison test the original integral must also **converge**.

10. It looks like this integral will probably diverge and so we'll need a smaller function that we know converges. Also, note that for $x > 1$ (i.e. the x 's in our interval of integration) we will have $\ln(x) > 0$.

Also using the hint we can see that $x > \ln(x)$ and because we have $x > 1$ we also know that $x^2 > x$.

Putting all of this together we get that $x^2 > \ln(x)$ in the range of x we're working on. This means that if we drop the logarithm from the denominator the denominator WILL get larger. So,

$$\begin{aligned} \frac{x+1}{x^2 - \ln(x)} &\geq \frac{x}{x^2 - \ln(x)} && \text{b/c } x+1 > x \\ &\geq \frac{x}{x^2 - 0} && \text{by reasoning above...} \\ &= \frac{1}{x} \end{aligned}$$

Now, we know that $\int_2^{\infty} \frac{1}{x} dx$ will diverge because $p = 1 \leq 1$ and so by the comparison test the original integral must also **diverge**.

11. For this problem we have $f(x) = \ln(1 + e^{3x})$ and $\Delta x = \frac{1}{2}$.

MidPoint Rule

$$\int_4^6 \frac{\cos x}{x} dx \approx \frac{1}{2} [f(4.25) + f(4.75) + f(5.25) + f(5.75)] = 0.075134$$

Trapezoid Rule

$$\int_4^6 \frac{\cos x}{x} dx \approx \frac{1}{4} [f(4) + 2f(4.5) + 2f(5) + 2f(5.5) + f(6)] = 0.068524$$

Simpson's Rule

$$\int_4^6 \frac{\cos x}{x} dx \approx \frac{1}{6} [f(4) + 4f(4.5) + 2f(5) + 4f(5.5) + f(6)] = 0.073017$$

For comparison's sake,

$$\int_4^6 \frac{\cos x}{x} dx = 0.072924$$