3. (2 pts) $\dfrac{dx}{dt} = -(2t+2)\mathbf{e}^{t^2+2t}$ $\quad \dfrac{dy}{dt} = 3t^2 - 4t - 8$ $\quad \dfrac{dy}{dx} = \dfrac{3t^2-4t-8}{-(2t+2)\mathbf{e}^{t^2+2t}}$

We need to find the *t* (or *t*'s) that give the point in question.

$$2 = 3 - \mathbf{e}^{t^2+2t} \quad \to \quad \mathbf{e}^{t^2+2t} = 1 \quad \to \quad t = 0, t = -2$$

$$4 = t^3 - 2t^2 - 8t + 4 \quad \to \quad t^3 - 2t^2 - 8t = t(t-4)(t+2) = 0 \quad \to \quad t = 0, -2, 4$$

So, we have two *t*'s to work with here.

$$t = 0: m = \left.\frac{dy}{dx}\right|_{t=0} = \frac{-8}{-2} = 4 \qquad \boxed{y = 4 + 4(x-2) = 4x - 4}$$

$$t = -2: m = \left.\frac{dy}{dx}\right|_{t=-2} = \frac{12}{2} = 6 \qquad \boxed{y = 4 + 6(x-2) = 6x - 8}$$

4. (3 pts) Note that we completely described the curve/path that this particle travels in Set 4, Problem 10. In that problem we saw that the particle will trace out once in $\frac{1}{8}\pi \le t \le \frac{3}{8}\pi$ and will trace out 92 times. To do this problem all we need to do is determine the length of a since trace (which will be the length of the curve) and then for the total distance traveled we can just multiple the length by the number of traces. So

$$\frac{dx}{dt} = 16\sin(4t)\cos(4t) \qquad \frac{dy}{dt} = 8\cos(4t)$$

$$ds = \sqrt{256\sin^2(4t)\cos^2(4t) + 64\cos^2(4t)} = 8\left|\cos(4t)\right|\sqrt{4\sin^2(4t)+1} = -8\cos(4t)\sqrt{4\sin^2(4t)+1}$$

Note that we can drop the absolute value bars provided we also add in a minus sign because when $\frac{1}{8}\pi \le t \le \frac{3}{8}\pi$ we have $\frac{1}{2}\pi \le 4t \le \frac{3}{2}\pi$ and so $\cos(4t) \le 0$. The length of the curve is then,

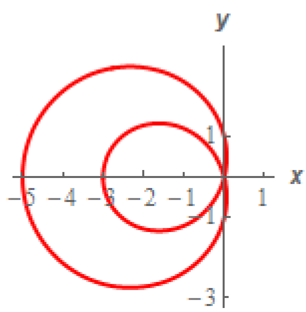
$$L = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} -8\cos(4t)\sqrt{4\sin^2(4t)+1}\,dt$$

This requires the trig substitution $\sin(4t) = \frac{1}{2}\tan\theta$ and I'll leave it to you to verify that this will give,

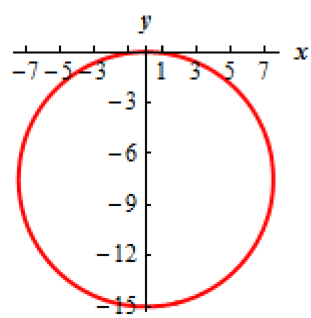
$$L = -2\int_{\tan^{-1}(2)}^{\tan^{-1}(-2)} \sec^3\theta\,d\theta = -\left(\sec\theta\tan\theta + \ln\left|\sec\theta + \tan\theta\right|\right)\Big|_{\tan^{-1}(2)}^{\tan^{-1}(-2)} = \boxed{5.9158}$$

So, the length of the curve is then 4.5912 and the distance traveled is $92 \times (5.9158) = \boxed{544.2536}$

7. (2 pts)



8. NOT GRADED



9. (3 pts) We first need to determine "when" the curve goes through the origin so,

$$1-4\cos\theta=0 \qquad \rightarrow \qquad \theta=\cos^{-1}\left(\tfrac{1}{4}\right)=1.3181$$

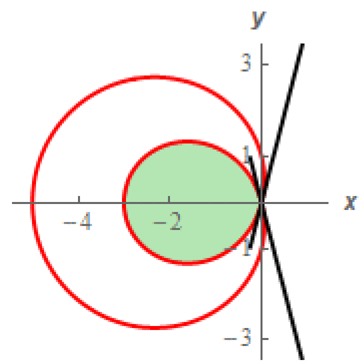
This is actually the second angle we need. We are at the (Cartesian) point (-3,0) when $\theta=0$ and so the angle from the calculator is after this and so is the angle where we come back out of the inner loop and so is the second angle. The first angle is then simple negative of this angle or -1.3181.

A sketch is to the right. The area is,

$$A=\tfrac{1}{2}\int_{-1.3181}^{1.3181}(1-4\cos\theta)^2\,d\theta=\tfrac{1}{2}\int_{-1.3181}^{1.3181}1-8\cos\theta+16\cos^2\theta\,d\theta$$

$$=\tfrac{1}{2}\int_{-1.3181}^{1.3181}9-8\cos\theta+8\cos(2\theta)\,d\theta$$

$$=\tfrac{1}{2}\left(9\theta+8\sin\theta+4\sin(2\theta)\right)\Big|_{-1.3181}^{1.3181}=\boxed{6.0536}$$



Not Graded

1.

$$\frac{dy}{dt}=24t^2+18t-1 \qquad \frac{dx}{dt}=10t-1 \qquad \boxed{\frac{dy}{dx}=\frac{24t^2+18t-1}{10t-1}}$$

$$\frac{d}{dt}\left(\frac{dy}{dx}\right)=\frac{(48t+18)(10t-1)-10(24t^2+18t-1)}{(10t-1)^2}=\frac{240t^2-48t-8}{(10t-1)^2}$$

$$\frac{d^2y}{dx^2}=\frac{\frac{240t^2-48t-8}{(10t-1)^2}}{10t-1}=\boxed{\frac{240t^2-48t-8}{(10t-1)^3}}$$

2.

$$\frac{dx}{dt}=\cos\left(\tfrac{\pi}{2}t\right)-\tfrac{\pi}{2}\sin\left(\tfrac{\pi}{2}t\right) \qquad \frac{dy}{dt}=2t \qquad \frac{dy}{dx}=\frac{2t}{\cos\left(\frac{\pi}{2}t\right)-\frac{\pi}{2}\sin\left(\frac{\pi}{2}t\right)}$$

$$m=\left.\frac{dy}{dx}\right|_{t=4}=\frac{8}{1}=8 \qquad x=4\cos(2\pi)=4, \qquad y=4^2-3=13 \qquad \boxed{y=13+8(x-4)=8x-19}$$

Find the equation of the tangent line to the given parametric curve at the point corresponding to the given value of the parameter.

$$x=t\cos\left(\tfrac{\pi}{2}t\right), \quad y=t^2-3 \quad \text{at} \quad t=4$$

5.

$$\frac{dy}{dx} = \cos(4t) - 4t\sin(4t) \qquad \frac{dy}{dt} = 2t + 6$$

$$A = \int 2\pi x\,ds = \boxed{2\pi\int_0^4 t\cos(4t)\sqrt{\left(\cos(4t) - 4t\sin(4t)\right)^2 + \left(2t+6\right)^2}\,dt}$$

6. First we'll need to use a half angle formula to get the trig function into a form we can deal with.

$$\tfrac{1}{2}\left(1+\cos(\theta)\right) = r \quad \to \quad \tfrac{1}{2}r + \tfrac{1}{2}r\cos(\theta) = r^2 \quad \Rightarrow \quad \boxed{\tfrac{1}{2}\sqrt{x^2+y^2} + \tfrac{1}{2}x = x^2 + y^2}$$

10. We'll start off with the intersection points.

$$4\sin\theta = 2 - \sin\theta \qquad 5\sin\theta = 2$$

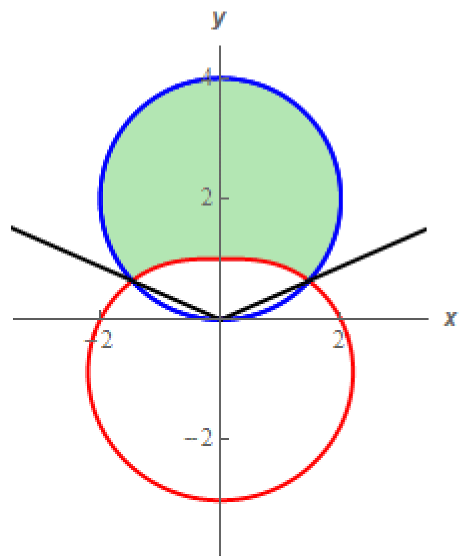
$$\theta = \sin^{-1}\left(\tfrac{2}{5}\right) = 0.4115 \qquad \theta = \pi - 0.4115 = 2.7301$$

The area is then,

$$A = \tfrac{1}{2}\int_{0.4115}^{2.7301}\left(4\sin\theta\right)^2 - \left(2-\sin\theta\right)^2\,d\theta$$

$$= \tfrac{1}{2}\int_{0.4115}^{2.7301}\tfrac{7}{2} + 4\sin\theta - \tfrac{15}{2}\cos(2\theta)\,d\theta$$

$$= \tfrac{1}{2}\left(\tfrac{7}{2}\theta - 4\cos\theta - \tfrac{15}{2}\sin(2\theta)\right)\Big|_{0.4115}^{2.7301} = \boxed{10.4731}$$



11. In this case we want the portion of the area inside the upper figure that we didn't compute in **#10** and note that this cannot be done as a single integral. So, the complete area of the upper figure is,

$$A = \pi(2)^2 = 4\pi$$

It is just a circle after all….

The area we're after is then,

$$\text{Area} = 4\pi - 10.4731 = \boxed{2.0933}$$