4. (2 pts) To do this one we'll need to put absolute value bars on the sequence terms to deal with the alternating sign.

$$\lim_{n\to\infty}\left|\frac{(-1)^n\left(8-n^2\right)}{9n+7n^3}\right| = \lim_{n\to\infty}\frac{8-n^2}{9n+7n^3} = 0$$

So, because this limit is zero (and only because it is zero) we know from the fact given in class that,

$$\lim_{n\to\infty}\frac{(-1)^n\left(8-n^2\right)}{9n+7n^3} = 0$$

Therefore, the sequence **converges** to 0.

6. (2 pts) We'll need the increasing/decreasing information first here and that will require some Calc I.

$$f(x) = \frac{5+x}{5,250,000+10x^2} \qquad\qquad f'(x) = \frac{5,250,000-100x-10x^2}{\left(5,250,000+10x^2\right)^2}$$

I'll leave it to you to verify that we get two critical points $x = -729.5861$ and $x = 719.5861$ and that the derivative is positive in the range $1 \le x \le 719.5861$ and negative in the range $x \ge 719.5861$ and so the function will both increase and decrease for $x \ge 1$ and so the sequence is **not** monotonic.

For bounds we can see that all the sequence terms are positive and so the sequence will be **bounded below** by 0. Also, from our increasing/decreasing work above we know that the function will increase until 719.5861 and then decrease. This means that either $a_{719} = 0.0000694843665$ or $a_{720} = 0.000069484378$ must be the largest sequence term. Or we could also note that all terms must be less than the maximum which will occur at 719.5861 (or, $f(719.5861) = 0.0000694843894$). From this we can see that the sequence will be **bounded above** by 0.0000694843894 or $a_{720} = 0.000069484378$ (or any larger number of course). The sequence is therefore **bounded**.

10. (2 pts) $\lim\limits_{n\to\infty} 13 = 13 \ne 0$. The series terms are not zero in the limit and so by the **Divergence Test** the series will **diverge**.

11. (2 pts) This is a geometric series. The series starts at $n = 1$ so we'll need all the exponents to be *n*-1.

$$\sum_{n=1}^{\infty}\frac{(-3)^{1-2n}}{4^{2-n}} = \sum_{n=1}^{\infty}\frac{4^{n-2}}{(-3)^{2n-1}} = \sum_{n=1}^{\infty}\frac{4^{n-1}\left(4^{-1}\right)}{9^n(-3)^{-1}} = \sum_{n=1}^{\infty}\frac{4^{n-1}\left(4^{-1}\right)}{9^{n-1}9^1(-3)^{-1}} = \sum_{n=1}^{\infty}\frac{-3}{4(9)}\left(\frac{4}{9}\right)^{n-1} = \sum_{n=1}^{\infty}\frac{-1}{12}\left(\frac{4}{9}\right)^{n-1}$$

So, we have $|r| = \left|\frac{4}{9}\right| = \frac{4}{9} < 1$ and so the series will **converge** and its value is,

$$\sum_{n=1}^{\infty}\frac{(-3)^{1-2n}}{4^{2-n}} = \frac{-\frac{1}{12}}{1-\left(\frac{4}{9}\right)} = \boxed{-\frac{3}{20}}$$

14. (2 pts) This looks like a telescoping series. So set up the partial sums and I'll leave the partial fraction details to you to verify.

$$
\begin{aligned}
s_N &= \sum_{n=4}^{N} \frac{6}{n^2-4n+3} = \sum_{n=4}^{N} \frac{3}{n-3} - \frac{3}{n-1} \\
&= \left(\tfrac{3}{1} - \cancel{\tfrac{3}{3}}\right) + \left(\tfrac{3}{2} - \cancel{\tfrac{3}{4}}\right) + \left(\cancel{\tfrac{3}{3}} - \cancel{\tfrac{3}{5}}\right) + \left(\cancel{\tfrac{3}{4}} - \cancel{\tfrac{3}{6}}\right) + \left(\cancel{\tfrac{3}{5}} - \cancel{\tfrac{3}{7}}\right) \cdots \\
&\quad \cdots + \left(\cancel{\tfrac{3}{N-7}} - \cancel{\tfrac{3}{N-5}}\right) + \left(\cancel{\tfrac{3}{N-6}} - \cancel{\tfrac{3}{N-4}}\right) + \left(\cancel{\tfrac{3}{N-5}} - \cancel{\tfrac{3}{N-3}}\right) + \left(\cancel{\tfrac{3}{N-4}} - \tfrac{3}{N-2}\right) + \left(\cancel{\tfrac{3}{N-3}} - \tfrac{3}{N-1}\right) \\
&= 3 + \tfrac{3}{2} - \tfrac{3}{N-2} - \tfrac{3}{N-1} = \tfrac{9}{2} - \tfrac{3}{N+3} - \tfrac{3}{N+4}
\end{aligned}
$$

Note that the only way to know you've got all the correct terms is to keep listing them out until you get all the terms to cancel and then you know everything from that point on will also cancel. You'll also need to do this at the "end" of the partial sum.

We can now take the limit of the partial sums.

$$\lim_{n\to\infty} s_N = \lim_{n\to\infty} \left(\tfrac{9}{2} - \tfrac{3}{N+3} - \tfrac{3}{N+4}\right) = \tfrac{9}{2}$$

So, the sequence of partial sums converges and so the series will also **converge** and the value of the series is $\frac{9}{2}$.

Not Graded

1. Sequence **converges** to $-\frac{10}{13}$

$$\lim_{n\to\infty} \frac{7n+3n^2-10n^4}{1+n^3+13n^4} = -\frac{10}{13}$$

2. In order to compute the limit here we need to combine the two logs prior to taking the limit.

$$\lim_{n\to\infty} \ln\left(\frac{4+25n}{27+3n}\right) = \ln\left(\frac{25}{3}\right)$$

The sequence converges to $\ln\left(\frac{25}{3}\right)$.

3. To do this one we need to write out a few of the terms.

$$\left\{\cos(n\pi)\right\}_0^\infty = \left\{1,-1,1,-1,1,-1,\ldots\right\} = \left\{(-1)^n\right\}_0^\infty$$

So, this sequence "oscillates" between 1, and -1 and so $\lim_{n\to\infty} \cos(n\pi)$ will not exist and so the sequence **diverges.**

5. Let's first get the increasing/decreasing information. To do this we'll need to do some Calc I.

$$f(x) = \frac{2-3x}{1-x} \qquad\qquad f'(x) = -\frac{1}{(x-1)^2}$$

We can see that the derivative will always be negative for $x \geq 2$ and therefore the function, and hence the sequence, will be decreasing. The sequence is therefore also **monotonic**. Now, because we know that the sequence is decreasing the first term in the sequence will be the largest and so the sequence will be **bounded above** by **4**. Now, also notice that for $n \geq 2$ both the numerator and denominator are negative and so each sequence term will be positive and so the sequence is **bounded below** by 0. This also means that the sequence is **bounded**.

7.

(a) $\displaystyle\sum_{n=1}^{\infty}\frac{1+n^2}{1+n^4} = \frac{2}{2}+\frac{5}{17}+\frac{10}{82}+\sum_{n=4}^{\infty}\frac{1+n^2}{1+n^4} = \frac{987}{697}+\sum_{n=4}^{\infty}\frac{1+n^2}{1+n^4} = 1.41607+\sum_{n=4}^{\infty}\frac{1+n^2}{1+n^4}$

(b) $\displaystyle 1.70701 = 1.41607+\sum_{n=4}^{\infty}\frac{1+n^2}{1+n^4} \quad\Rightarrow\quad \boxed{\sum_{n=4}^{\infty}\frac{1+n^2}{1+n^4} = 0.29094}$

8.

(a) $\displaystyle\lim_{n\to\infty}\frac{2n^4-9n^2+1}{7n^4+n^3+n^2+4} = \frac{2}{7}$ so the sequence **converges** (and will converge to $\frac{2}{7}$)

(b) From the work above we see that the limit of the series terms is not zero and so by the **Divergence Test** the series will **diverge.**

9. $\displaystyle\lim_{n\to\infty}\frac{7n+\mathbf{e}^{-n}}{4+5n} = \frac{7}{5}$. So, because the sequence of partial sums converge the series will also **converge** and the value of the series is $\frac{7}{5}$.

12. This series only differs from that in **#11** by the starting value of the index and so will **converge** as well. To get its value all we need to do is strip out the first two terms from the series from **#11** and use the value we found there.

$$-\frac{3}{20} = \sum_{n=1}^{\infty}\frac{(-3)^{1-2n}}{4^{2-n}} = -\tfrac{1}{12}-\tfrac{1}{27}+\sum_{n=4}^{\infty}\frac{(-3)^{1-2n}}{4^{2-n}} \quad\Rightarrow\quad \boxed{\sum_{n=3}^{\infty}2^{2-3n}(-3)^{1+n} = -\tfrac{4}{135}}$$

13. This is a **harmonic series** (we can always factor the $-\frac{9}{10}$ out of the series and the starting value isn't material) and so the series **diverges**.