Math 3301 **Homework Set 1** **10 Points**

Direction Fields

Sketch the direction fields for each of the following differential equations. Based on the direction field, determine the behavior of the solution, $y(t)$, as $t \to \infty$. If this behavior depends on the value of $y(0)$ give this dependence.

1. $\dfrac{dy}{dx} = 6y^2\left(y^2 - 4y - 12\right)$

2. $y' = (y-4)\left(1 - \mathbf{e}^{5y-10}\right)$

Linear Differential Equations

For problems 3 & 4, solve the given IVP.

3. $\left(x^2+2\right)y' = 4xy + \left(x^2+2\right)^3 \cos(x) \qquad y(0) = -3$

4. $2t\,y' - (8-12t)\,y = t^8\mathbf{e}^{-6t} - 16t^6\mathbf{e}^{2t} \quad y(1) = \mathbf{e}^{-6}, \qquad t > 0$

5. It is known that the solution to the following differential equation will have a relative maximum at $t=1$. Assume that the solution and its derivative are continuous and determine the value of the solution at $t=1$. Note that because you don't have an initial condition you can't actual solve this differential equation. It is still possible to answer the question however.

$$y' - 2y = 4 - \mathbf{e}^{3t}$$

Hint : Recall from Calc I where relative extrema may occur and don't forget the differential equation (just because you can't solve it doesn't mean it isn't needed)!

6. Find the solution to the following IVP in terms of y_0. Determine the value of y_0 for which the solution to the following IVP will just touch, but not cross the *t*-axis.

$$2y' + 4y = 2 - 3t \qquad\qquad y(0) = y_0$$

Hint : Think of all the ways this can happen. Once you've got that figured out and the solution in terms of y_0 if there was just any way (cough, cough, #5…) of determining when this situation occurs the rest should be pretty easy (although you may need to use a calculator to solve an equation at some point).

7. Find the solution to the following IVP in terms of α. Find all possible long term behaviors of the solution (*i.e.* as $t \to \infty$). If this behavior depends on the value of α clearly give this dependence.

$$y' - 5y = 7 - 2\mathbf{e}^{2t} \qquad y(0) = \alpha^2 - 4$$