Separable Differential Equations

For problems 1 & 2 find the solution to the given IVP and determine (at least approximately) the interval of validity for the solution. Any approximate answers must be to at least the 4th decimal place and you may need computational aids in finding the interval of validity.

1.
$$(t^2 - 1)^2 y' = 6y^2 t$$
 $y(0) = 7$

2.
$$y' = \frac{3+4x}{2y+5}$$
 $y(0) = -2$

3. Solve the following IVP and determine the maximum value of the solution for t > 0. Any approximate answers must be to at least the 4th decimal place. You may assume that the solution will be positive for this problem.

 $y' = y\left(4t - t^2\right) \qquad \qquad y(0) = \mathbf{e}^3$

<u>Modeling, Part I</u>

For problems 4 & 5 you MUST set up and solve the appropriate IVP(s) using the solution to the IVP to find the answer to the question. If you use any decimals in your work then use at least 4 decimal places for each number.

4. A 1000 liter tank contains 400 liter of salt water with 20 grams of a salt dissolved in it. Salt water with a concentration of $c(t) = \frac{t}{10}e^{\frac{1-t}{30}}$ grams/liter flows in at a rate of 2 liter/min and a well mixed solution flows out at a rate of 2 liter/min. Determine how

much dye is in the water after 1 hour of operation.

5. A 600 gallon tank initially contains 400 gallons of water with 10 ounces of a contaminate dissolved in it. Contaminated water with a concentration of 8 ounces/gal flows into the tank at a rate of 3 gal/hour and a well mixed solution flows out at a rate of 5 gal/hour. This will continue 100 hours and at this point the concentration of the contaminated water is decreased to 6 ounce/gal and the flow rate in increases to 4 gal/hr and the well mixed solution exits the tank at a reduced rate of 2 gal/hr. At what time will the amount of contaminate reach 3000 ounces?

6. Take the situation from #6. Once the contaminate reaches 3000 ounces the outflow is increased to 4 gal/hr and all other quantities remain the same. Set up an IVP, but do not solve it, that will give the amount of contaminate in the tank for this new time frame.