

Modeling, Part II

For problems 1 & 2 you MUST set up and solve the appropriate IVP(s) using the solution to the IVP to find the answer to the question. If you use any decimals in your work then use at 4 decimal places for each number.

1. In a field the population of rabbits grows at a rate proportional to its population. There are originally 100 rabbits in the field and in the absence of outside factors there would be 200 rabbits in one months time (assume 4 weeks/month). Each week predators eat 21 rabbits and 6 rabbits migrate into the area. After 6 weeks the predators start eating 32 rabbits per week and 10 rabbits now migrate into the area. Will the rabbits survive to implement their plan for world domination? If they don't when did we finally put a stop to their evil scheme (i.e. when do they all die)?
2. A 10 kg mass is dropped off a bridge with an initial velocity of 50 cm/sec downward. If the air resistance on the object is given by $4v$ and the bridge is 25 meters above a lake how long does it take for the mass to hit the lake?
3. Take the situation from #2. Once the mass hits the lake the "air" resistance increases to $12v$. If the mass falls for a further 4 seconds before hitting the bottom of the lake, how deep is the lake?

Equilibrium Solutions

Find and classify each of the equilibrium points.

4. $y' = y(2y + 5)(y - 3)^2$
5. $y' = by + 3y^2$, $b < 0$

Euler's Method

6. Find an approximate value to the solution of the IVP below at $t = 0.2$ using both $h = 0.1$ and $h = 0.05$. All decimal work should be to at least the 4th decimal place.

$$y' - (t + e^{-2y})^2 = 1, \quad y(0) = -1$$