#1. (4 pts) Here are the three (don't forget the one for finding *r*) IVP's that we'll be needing.

$$P' = rP \qquad P(0) = 100 \qquad P(4) = 200$$

$$P_1' = rP_1 + 6 - 21 = rP_1 - 15 \qquad P_1(0) = 100 \qquad 0 \le t \le 6$$

$$P_2' = rP_2 + 10 - 32 = rP_2 - 22 \qquad P_2(0) = P_1(6) \quad t \ge 6$$

Note that we're using *t* in weeks so we had to use $t = 4$ for the one month population.

We first need to find *r* and this is an easy linear or separable differential equation to solve so I'll leave the details to you. Solving gives,

$$P(t) = 100\mathbf{e}^{rt}$$

Plugging in $t = 8$ and solving for *r* gives,

$$200 = 100\mathbf{e}^{r(4)} \;\Rightarrow\; 2 = \mathbf{e}^{r(4)} \;\Rightarrow\; r = \tfrac{1}{4}\ln(2)$$

The first IVP is then,

$$P_1' = \tfrac{1}{4}\ln(2)P_1 - 15 \qquad P_1(0) = 100$$

Here is the solution to this differential equation.

$$P_1' - \tfrac{1}{4}\ln(2)P_1 = -15 \qquad \mu(t) = \mathbf{e}^{-\frac{1}{4}\ln(2)t}$$

$$\int \left(\mathbf{e}^{-\frac{1}{4}\ln(2)t}P_1\right)' dt = \int -15\mathbf{e}^{-\frac{1}{4}\ln(2)t}\, dt$$

$$\mathbf{e}^{-\frac{1}{4}\ln(2)t}P_1 = \frac{60}{\ln(2)}\mathbf{e}^{-\frac{1}{4}\ln(2)t} + c$$

$$P_1(t) = \frac{60}{\ln(2)} + c\mathbf{e}^{-\frac{1}{4}\ln(2)t}$$

Plugging in the initial condition gives $c = 13.4383$ so the population during this time is,

$$P_1(t) = \frac{60}{\ln(2)} + 13.4383\mathbf{e}^{\frac{1}{4}\ln(2)t}$$

Now that we have this out of the way we can proceed with the second time frame. Here is the IVP for this time frame.

$$P_2' = \tfrac{1}{4}\ln(2)P_2 - 22 \qquad P_2(6) = P_1(6) = 124.5709$$

Solving this is identical to solving the first one and so I'll leave the details of the solution to you. Here is the population during this time frame.

$$P_2(t) = \frac{88}{\ln(2)} - 0.8437\mathbf{e}^{\frac{1}{4}\ln(2)t}$$

Okay, the coefficient of the exponential is negative and so eventually the population will reach zero and the world will be safe from the rabbits. The world will become safe at,

$$0 = \frac{88}{\ln(2)} - 0.8437\mathbf{e}^{\frac{1}{4}\ln(2)t} \qquad \Rightarrow \qquad t = 28.9338 \text{ weeks}$$

<u>#2.</u> (4 pts) I'll be working this assuming that downwards is the positive direction for all quantities. I'll also be defining the level of the bridge to be $s = 0$. Under these assumptions the IVP that governs this falling body is,

$$v' = 9.8 - \tfrac{4}{10} v = 9.8 - \tfrac{2}{5} v \qquad\qquad v(0) = 0.5$$

This is a simple linear differential equation to solve so I'll leave the solution details to you. The velocity at any time is,

$$v(t) = 24.5 - 24\mathbf{e}^{-\frac{2}{5}t}$$

Next we'll need the height function and this we find as follows,

$$s(t) = \int v(t)\,dt = \int 24.5 - 24\mathbf{e}^{-\frac{2}{5}t}\,dt = 24.5t + 60\mathbf{e}^{-\frac{2}{5}t} + c$$

Using the fact that $s(0) = 0$ by assumption we get that,

$$s(t) = 24.5t + 60\mathbf{e}^{-\frac{2}{5}t} - 60$$

To determine how long it took for the mass to hit the lake we need to solve,

$$25 = 24.5t + 60\mathbf{e}^{-\frac{2}{5}t} - 60 \qquad \Rightarrow \qquad \cancel{t = 2.01608} \qquad t = 2.60579$$

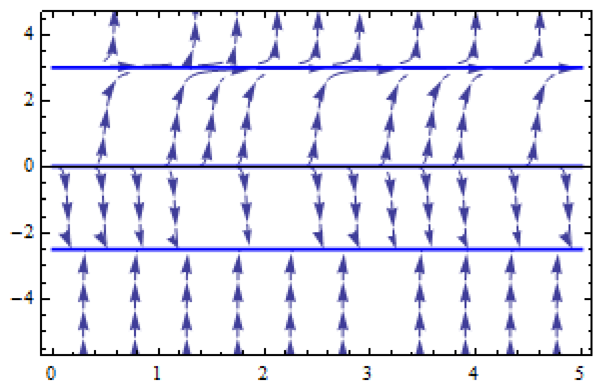
So, it took 2.60579 seconds for the mass to hit the lake and the velocity at which it hits the lake (which we'll need for **#3**) is $v(2.60579) = 16.0367$.

<u>#4.</u> (2 pts) So, the equilibrium solutions are,

$$y = 0, \quad y = 3, \quad y = -\tfrac{5}{2}$$

Here's a sketch of a few solutions for this differential equation. From this we can see that

$y = 3$	semi-stable
$y = 0$	unstable
$y = -\frac{5}{2}$	asymptotically stable



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<u>#3.</u> In this case the IVP will be,

$$v' = 9.8 - \tfrac{12}{10} v = 9.8 - \tfrac{6}{5} v \qquad\qquad v(2.60579) = 16.0367$$

I'll leave the solution details to you since this is a simple linear differential equation.

$$v(t) = 8.16667 + 179.47256\mathbf{e}^{-\frac{6}{5}t}$$

Using $s(2.60579) = 25$ the height function is,

$$s(t) = 8.16667t - 149.56047\mathbf{e}^{-\frac{6}{5}t} + 10.27766$$

The depth of the lake is then,

$$s(2.60579+4)-25=64.17105-25=\boxed{39.17105}$$

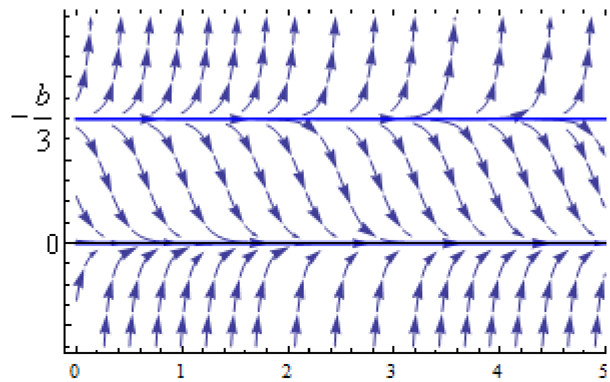
The lake is 39.17105 meters deep. Note that because I used all "zero" quantities at the bridge I had to add in the time it took to reach the lake and then subtract off the height of the bridge.

#5. Upon factoring the right side we get : $y' = y(b+3y)$

So, the equilibrium solutions are,

$$y=0, \quad y=-\frac{b}{3}>0$$

Do NOT get excited that you don't know what *b* is. It is a negative number and so the equilibrium solution is positive. If you need to think of it as $b=-6$ and you can see what's going on. Here's a sketch of a few solutions for this differential equation. From this we can see that



$y=-\frac{b}{3}$ unstable

$y=0$ asymptotically stable

#6. There really isn't much to do here other than run through the formulas and noting that,

$$f(t,y)=1+\left(t+\mathbf{e}^{-2y}\right)^2$$

Here are the results for $h=0.1$. The exact values are included for comparison purposes.

t	0.1	0.2
f_n	55.59815003	1.010021911
Approx	4.559815003	4.660817194
Exact	−0.151461103	0.094366657

Here are the results using $h=0.05$.

t	0.05	0.1	0.15	0.2
f_n	55.59815003	1.006153475	1.015805881	1.030011046
Approx	1.779907502	1.830215176	1.881005470	1.932506022
Exact	−0.348518328	−0.151461103	−0.016123087	0.094366657

So, it looks like for $h = 0.1$ we have $y(0.2) \approx 4.660817194$ and for $h = 0.05$ we have $y(0.2) \approx 1.932506022$. Note that the results are not all the great either. It takes an $h = 0.001$ or so to start getting decent results…..