#4. (4 pts) First, solve the differential equation.

$$r^2+10r-39=(r+13)(r-3)=0 \quad\Rightarrow\quad r_1=-13, r_2=3 \quad\Rightarrow\quad \underline{y(t)=c_1\mathbf{e}^{-13t}+c_2\mathbf{e}^{3t}}$$

$$\begin{aligned} c_1+c_2&=1-8\beta \\ -13c_1+3c_2&=4\beta^2 \end{aligned} \quad\rightarrow\quad \begin{aligned} c_1&=\tfrac{3-24\beta-4\beta^2}{16} \\ c_2&=\tfrac{13-104\beta+4\beta^2}{16} \end{aligned} \quad\Rightarrow\quad \boxed{y(t)=\tfrac{3-24\beta-4\beta^2}{16}\mathbf{e}^{-13t}+\tfrac{13-104\beta+4\beta^2}{16}\mathbf{e}^{3t}}$$

Now, in order for the solution to remain finite the second term must not be present. This means that the coefficient of this term must be zero which will happen if

$$\boxed{\beta=\tfrac{1}{2}\left(26\pm\sqrt{663}\right)=0.1256,\ \ 25.8744}$$

#6. (3 pts) First solve the differential equation.

$$r^2+4r+68=0 \quad\Rightarrow\quad r_{1,2}=-2\pm 8i \quad\Rightarrow\quad \underline{y(t)=c_1\mathbf{e}^{-2t}\cos(8t)+c_2\mathbf{e}^{-2t}\sin(8t)}$$

$$\begin{aligned} c_1&=3 \\ -2c_1+8c_2&=0 \end{aligned} \quad\rightarrow\quad \begin{aligned} c_1&=3 \\ c_2&=\tfrac{3}{4} \end{aligned} \quad\Rightarrow\quad \boxed{y(t)=3\mathbf{e}^{-2t}\cos(8t)+\tfrac{3}{4}\mathbf{e}^{-2t}\sin(8t)}$$

#7. (3 pts)

$$25r^2+10r+1=(5r+1)^2=0 \quad\Rightarrow\quad r_{1,2}=-\tfrac{1}{5} \quad\Rightarrow\quad \underline{y(t)=c_1\mathbf{e}^{-\frac{1}{5}t}+c_2t\mathbf{e}^{-\frac{1}{5}t}}$$

$$\begin{aligned} c_1\mathbf{e}^{-\frac{1}{5}}+c_2\mathbf{e}^{-\frac{1}{5}}&=2 \\ -\tfrac{1}{5}c_1\mathbf{e}^{-\frac{1}{5}}+\tfrac{4}{5}c_2\mathbf{e}^{-\frac{1}{5}}&=-3 \end{aligned} \quad\rightarrow\quad \begin{aligned} c_1&=\tfrac{23}{5}\mathbf{e}^{\frac{1}{5}} \\ c_2&=-\tfrac{13}{5}\mathbf{e}^{\frac{1}{5}} \end{aligned} \quad\Rightarrow\quad \boxed{y(t)=\tfrac{23}{5}\mathbf{e}^{-\frac{1}{5}(t-1)}-\tfrac{13}{5}t\mathbf{e}^{-\frac{1}{5}(t-1)}}$$

Not Graded

#1.

$$4r^2+r-11=0 \quad\Rightarrow\quad r_{1,2}=\tfrac{-1\pm\sqrt{177}}{8}=-1.788,\ 1.538 \quad\Rightarrow\quad \boxed{y(t)=c_1\mathbf{e}^{-1.788t}+c_2\mathbf{e}^{1.538t}}$$

#2.

$$2r^2-17r+21=(2r-3)(r-7)=0 \quad\Rightarrow\quad r_1=\tfrac{3}{2}, r_2=7 \quad\Rightarrow\quad \underline{y(t)=c_1\mathbf{e}^{\frac{3}{2}t}+c_2\mathbf{e}^{7t}}$$

$$\begin{aligned} c_1+c_2&=-4 \\ \tfrac{3}{2}c_1+7c_2&=1 \end{aligned} \quad\rightarrow\quad \begin{aligned} c_1&=-\tfrac{58}{11} \\ c_2&=\tfrac{14}{11} \end{aligned} \quad\Rightarrow\quad \boxed{y(t)=-\tfrac{58}{11}\mathbf{e}^{\frac{3}{2}t}+\tfrac{14}{11}\mathbf{e}^{7t}}$$

#3.

$$r^2+3r=r(r+3)=0 \quad\Rightarrow\quad r_1=0, r_2=-3 \quad\Rightarrow\quad \underline{y(t)=c_1+c_2\mathbf{e}^{-3t}}$$

$$\begin{aligned} c_1+c_2\mathbf{e}^{6}&=-11 \\ -3c_2\mathbf{e}^{6}&=1 \end{aligned} \quad\rightarrow\quad \begin{aligned} c_1&=-\tfrac{32}{3} \\ c_2&=-\tfrac{1}{3}\mathbf{e}^{-6} \end{aligned} \quad\Rightarrow\quad \boxed{y(t)=-\tfrac{32}{3}-\tfrac{1}{3}\mathbf{e}^{-3t-6}}$$

#5.

$$16r^2-96r+145=0 \quad\Rightarrow\quad r_{1,2}=3\pm\tfrac{1}{4}i \quad\Rightarrow\quad \underline{y(t)=c_1\mathbf{e}^{3t}\cos\left(\tfrac{1}{4}t\right)+c_2\mathbf{e}^{3t}\sin\left(\tfrac{1}{4}t\right)}$$

$$\begin{aligned} c_1\mathbf{e}^{24\pi}&=0 \\ 3\mathbf{e}^{24\pi}c_1+\tfrac{1}{4}\mathbf{e}^{24\pi}c_2&=5 \end{aligned} \quad\rightarrow\quad \begin{aligned} c_1&=0 \\ c_2&=20\mathbf{e}^{-24\pi} \end{aligned} \quad\Rightarrow\quad \boxed{y(t)=20\mathbf{e}^{3t-24\pi}\sin\left(\tfrac{1}{4}t\right)}$$

#8. First solve the differential equation

$$r^2+20r+100=(r+10)^2=0 \quad\Rightarrow\quad r_{1,2}=-10 \quad\Rightarrow\quad \underline{y(t)=c_1\mathbf{e}^{-10t}+c_2t\mathbf{e}^{-10t}}$$

$$\begin{aligned} c_1&=-5 \\ -10c_1+c_2&=7 \end{aligned} \quad\rightarrow\quad \begin{aligned} c_1&=-5 \\ c_2&=-43 \end{aligned} \quad\Rightarrow\quad \boxed{y(t)=-5\mathbf{e}^{-10t}-43t\mathbf{e}^{-10t}}$$