Fundamental Sets of Solutions

1. In the case of real, distinct roots ($r_1 \neq r_2$) I made the claim that the two solutions were $y_1(t) = \mathbf{e}^{r_1 t}$ and $y_2(t) = \mathbf{e}^{r_2 t}$. Show that these two solutions are a fundamental set of solutions and that the general solution in this case is in fact $y(t) = c_1\mathbf{e}^{r_1 t} + c_2\mathbf{e}^{r_2 t}$. Make sure that you clearly justify your answer.

2. Suppose that we know that $f(x) = x^2$ and that $W(f,g) = x^4\cos(3x)$. Determine the most general possible $g(x)$ that would have given this Wronskian. You may assume that $x > 0$ for this problem.

Undetermined Coefficients, Part I

For problems 4 – 7 use undetermined coefficients to determine the general solution to the given differential equation.

3. $9y'' - 6y' + 145y = 4\mathbf{e}^{-3t}$

4. $y'' - 20y' + 100y = 13\cos(2t)$

5. $4y'' + 12y' + 9y = 3t^2\mathbf{e}^{-t}$

6. $y'' + 4y = (50t - 5)\sin(3t)$

7. Solve the following IVP

$$y'' - 9y' + 18y = 72t^3 + 9, \qquad y(0) = 2 \quad y'(0) = -6$$