<u>#3.</u> (2 pts) First we need to write this in terms of Heaviside functions.

$$h(t) = 6t + (8-2t)u_4(t) + \left(10\mathbf{e}^{-t} - 8\right)u_8(t)$$
$$= 12 - 2(t-4)u_4(t) + \left(10\mathbf{e}^{-(t-8+8)} - 8\right)u_8(t)$$
$$= 12 - 2(t-4)u_4(t) + \left(-8 + 10\mathbf{e}^{-8}\mathbf{e}^{-(t-8)}\right)u_8(t)$$
$$\boxed{H(s) = \frac{12}{s} - \frac{2\mathbf{e}^{-4s}}{s^2} + \mathbf{e}^{-8s}\left(-\frac{8}{s} + \frac{10\mathbf{e}^{-8}}{s+1}\right)}$$

<u>#4.</u> (2 pts) No partial fractions with this one….

$$H(s) = \frac{3\mathbf{e}^{-s}}{s} + \mathbf{e}^{-10s}\frac{7\frac{4}{4}}{s^2+16} + \mathbf{e}^{-4s}\frac{s+7}{(s+7)^2+9}$$
$$\boxed{h(t) = 3u_1(t) + \tfrac{7}{4}u_{10}(t)\sin(4t-40) + u_4(t)\mathbf{e}^{-7t+28}\cos(3t-12)}$$

<u>#5.</u> (2 pts)

$$F(s) = \left(8 - 2\mathbf{e}^{-10s}\right)\frac{1}{(s+3)(s-7)} + 4\mathbf{e}^{-6s}\frac{s}{(s+3)(s-7)} = \left(8-2\mathbf{e}^{-10s}\right)G(s) + 4\mathbf{e}^{-6s}H(s)$$

Now take the inverse transform of each individual piece, I'll leave the details of the partial fractions to you to verify.

$$G(s) = \frac{\frac{1}{10}}{s-7} - \frac{\frac{1}{10}}{s+3} \quad \Rightarrow \quad \underline{g(t) = \tfrac{1}{10}\left(\mathbf{e}^{7t} - \mathbf{e}^{-3t}\right)}$$
$$H(s) = \frac{\frac{7}{10}}{s-7} + \frac{\frac{3}{10}}{s+3} \quad \Rightarrow \quad \underline{h(t) = \tfrac{1}{10}\left(7\mathbf{e}^{7t} + 3\mathbf{e}^{-3t}\right)}$$

The inverse transform is then,

$$F(s) = 8G(s) - 2\mathbf{e}^{-10s}G(s) + 4\mathbf{e}^{-6s}H(s)$$
$$\boxed{f(t) = 8g(t) - 2u_{10}(t)g(t-10) + 4u_6(t)h(t-6)}$$

where $g(t)$ and $h(t)$ are show above.

<u>#7.</u> (2 pts) Take the transform of everything.

$$s^2Y(s) - sy(0) - y'(0) - 6\left(sY(s) - y(0)\right) + 9Y(s) = \frac{7}{s-3}$$
$$\left(s^2 - 6s + 9\right)Y(s) + 8s - 47 = \frac{7}{s-3}$$
$$\left(s^2 - 6s + 9\right)Y(s) = \frac{7 + (47-8s)(s-3)}{s-3}$$
$$Y(s) = \frac{-8s^2 + 71s - 134}{(s-3)^3}$$

Now we need to do partial fractions (I'll leave the details to you).

$$Y(s) = \frac{A}{s-3} + \frac{B}{(s-3)^2} + \frac{C}{(s-3)^3} = \frac{-8}{s-3} + \frac{23}{(s-3)^2} + \frac{7\left(\frac{2}{2}\right)}{(s-3)^3}$$

$$\boxed{y(t) = -8\mathbf{e}^{3t} + 23t\mathbf{e}^{3t} + \tfrac{7}{2}t^2\mathbf{e}^{3t}}$$

#9. (2 pts) Take the transform of everything.

$$s^2Y(s) - sy(0) - y'(0) + 9Y(s) = \frac{-12}{s^2+9}$$

$$(s^2+9)Y(s) - 4s + 7 = \frac{-12}{s^2+9}$$

$$Y(s) = \frac{4s-7}{s^2+9} - \frac{12}{(s^2+9)^2}$$

Now we do inverse transforms and notice that we don't need to do partial fractions here.

$$Y(s) = \frac{4s-7}{s^2+9} - \frac{12}{(s^2+9)^2} = \frac{4s}{s^2+9} - \frac{7\left(\frac{3}{3}\right)}{s^2+9} - \frac{6(2)\frac{3^3}{3^3}s}{(s^2+9)^2}$$

$$\boxed{y(t) = 4\cos(3t) - \tfrac{7}{3}\sin(3t) - \tfrac{2}{9}\left[\sin(3t) - 3t\cos(3t)\right]}$$

Not Graded

#1. $f(t) = 13u_{12}(t)\mathbf{e}^{-\frac{1}{3}(t-12)} + 9u_7(t)\sinh(5(t-7)) \quad \Rightarrow \quad \boxed{F(s) = \frac{13\mathbf{e}^{-12s}}{s+\frac{1}{3}} + \frac{45\mathbf{e}^{-7s}}{s^2-25}}$

#2. We've got some manipulation of the functions here to do first.

$$g(t) = u_2(t)\sin(6(t-2+2)) - 8(t-4+4)^2u_4(t)$$

$$= u_2(t)\sin(6(t-2)+12) - 8\left((t-4)^2 + 8(t-4) + 16\right)^2 u_4(t)$$

In the first term it looks like we're shifting $g(t) = \sin(6t+12)$ and in the second it looks like we're shifting $h(t) = t^2 + 8t + 16$. The transform is then,

$$\boxed{G(s) = \mathbf{e}^{-2s}\,\frac{s\sin(12) + 6\cos(12)}{s^2+36} - 8\mathbf{e}^{-4s}\left(\frac{2}{s^3} + \frac{8}{s^2} + \frac{16}{s}\right)}$$

#6.

$$G(s) = \left(\mathbf{e}^{-5s} + 8\mathbf{e}^{-3s}\right)\frac{1}{2s^2+8} - 12\mathbf{e}^{-4s}\frac{1}{s\left(2s^2+8\right)} = \left(\mathbf{e}^{-5s} + 8\mathbf{e}^{-3s}\right)F(s) - 12\mathbf{e}^{-4s}H(s)$$

Now take the inverse transform of each individual piece, I'll leave the details of the partial fractions to you to verify.

$$H(s) = \frac{\frac{1}{8}}{s} - \frac{\frac{1}{4}s}{2s^2+8} = \frac{1}{8}\left(\frac{1}{s} - \frac{s}{s^2+4}\right) \quad \Rightarrow \quad \underline{h(t) = \tfrac{1}{8}\left(1-\cos(2t)\right)}$$

$$F(s) = \frac{1}{2s^2+8} = \frac{1}{2}\frac{1\left(\frac{2}{2}\right)}{s^2+4} \quad \Rightarrow \quad \underline{f(t) = \frac{1}{4}\sin(2t)}$$

The inverse transform is then,

$$G(s) = \mathbf{e}^{-5s}F(s) + 8\mathbf{e}^{-3s}F(s) - 12\mathbf{e}^{-4s}H(s)$$

$$\boxed{f(t) = u_5(t)f(t-5) + 8u_3(t)f(t-3) - 12u_4(t)h(t-4)}$$

where $h(t)$ is show above.

#8. Take the transform of everything.

$$4\left(s^2Y(s) - sy(0) - y'(0)\right) - 48\left(sY(s) - y(0)\right) + 145Y(s) = \frac{7}{s-1}$$

$$\left(4s^2 - 48s + 145\right)Y(s) - 12 = \frac{7}{s-1}$$

$$\left(4s^2 - 48s + 145\right)Y(s) = \frac{7 + 12(s-1)}{s-1}$$

$$Y(s) = \frac{12s-5}{(s-1)\left(4s^2-48s+145\right)}$$

Now we need to do partial fractions (I'll leave the details to you).

$$Y(s) = \frac{A}{s-1} + \frac{Bs+C}{4s^2-48s+145} = \frac{1}{101}\left(\frac{7}{s-1} + \frac{1}{4}\frac{-28s+1520}{s^2-12s+\frac{145}{4}}\right)$$

$$= \frac{1}{101}\left(\frac{7}{s-1} + \frac{-7(s-6+6)+380}{(s-6)^2+\frac{1}{4}}\right)$$

$$= \frac{1}{101}\left(\frac{7}{s-1} - \frac{7(s-6)}{(s-6)^2+\frac{1}{4}} + \frac{338\frac{2}{2}}{(s-6)^2+\frac{1}{4}}\right)$$

$$\boxed{y(t) = \tfrac{1}{101}\left(7\mathbf{e}^{t} - 7\mathbf{e}^{6t}\cos\left(\tfrac{t}{2}\right) + 676\mathbf{e}^{6t}\sin\left(\tfrac{t}{2}\right)\right)}$$