

IVP's with Step Functions

Use Laplace transforms to solve the given IVP. In the partial fraction stage all quadratics that can be factored with integer coefficients must be factored!

$$1. y'' - 4y' + 20y = 12 - u_4(t) + 8u_6(t) \quad y(0) = 3, y'(0) = 1$$

$$2. 2y'' - y' - 6y = 3u_6(t)e^{2t-12} \quad y(0) = 2, y'(0) = -3$$

$$3. y'' - 4y = 4t - 3(t-7)u_7(t) \quad y(0) = 0, y'(0) = 0$$

Dirac-Delta Function

Use Laplace transforms to solve the given IVP.

$$4. 9y'' - 30y' + 25y = \delta(t-2) \quad y(0) = 10, y'(0) = 0$$

$$5. y'' + 11y' + 18y = 6u_1(t) - 8\delta(t-7) \quad y(0) = 0, y'(0) = 0$$

Convolution Integrals

$$6. \text{ Find the Laplace Transform of } f(t) = \int_0^t \tau \sin(3\tau - 3t) \sin(9\tau) d\tau.$$

$$7. \text{ Use a convolution integral (make sure you evaluate the integral!) to find the inverse transform of } G(s) = \frac{24}{s(s^2 + 64)}.$$

$$8. \text{ Find the solution to the following IVP in terms of } g(t).$$

$$y'' - 6y' = g(t) \quad y(0) = 0, \quad y'(0) = 8$$