

**Eigenvalues and Eigenvectors**

Find the eigenvalues and eigenvectors for the given matrices.

$$1. A = \begin{bmatrix} 10 & 1 \\ -4 & 6 \end{bmatrix}$$

$$2. B = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ -37 & -\frac{1}{4} \end{bmatrix}$$

$$3. C = \begin{bmatrix} 4 & 7 \\ 1 & -2 \end{bmatrix}$$

**Systems of Differential Equations**

Convert the IVP into a system of differential equations and give your answer in matrix form.

$$4. 7y'' - 9y' + 21y = 0, \quad y(0) = 4, y'(0) = -6$$

$$5. y^{(4)} - 16y' + y = 0, \quad y(3) = 0, y'(3) = -8, y''(3) = -5, y'''(3) = 10$$

**Real, Distinct Eigenvalues**

6. Find the general solution to the following system

$$\vec{x}' = \begin{bmatrix} -9 & 5 \\ 4 & -3 \end{bmatrix} \vec{x}$$

For problems 7 solve the system, sketch the phase portrait for the system and determine the stability of the equilibrium solution.

$$7. \vec{x}' = \begin{bmatrix} 4 & 5 \\ 3 & 2 \end{bmatrix} \vec{x} \quad \vec{x}(0) = \begin{pmatrix} -8 \\ 7 \end{pmatrix}$$

8. Answer each of the following questions about the given IVP.

(a) Convert the IVP into a system.

(b) Solve the system and use this solution to find the solution to the original IVP.

(c) Sketch the phase portrait for the system and determine the stability of the equilibrium solution.

$$y'' - 8y' + 12y = 0 \quad y(0) = 3, \quad y'(0) = 1$$

Continued on Back  $\Rightarrow$

**Complex Eigenvalues**

Solve the system, sketch the phase portrait for the system and determine the stability of the equilibrium solution.

$$9. \vec{x}' = \begin{bmatrix} -3 & -13 \\ 1 & 3 \end{bmatrix} \vec{x} \quad \vec{x}(0) = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$$

$$10. \vec{x}' = \begin{bmatrix} 6 & 5 \\ -10 & -4 \end{bmatrix} \vec{x} \quad \vec{x}(0) = \begin{pmatrix} -5 \\ 0 \end{pmatrix}$$

**Repeated Eigenvalues**

Solve the system, sketch the phase portrait for the system and determine the stability of the equilibrium solution.

$$11. \vec{x}' = \begin{bmatrix} -10 & -4 \\ 9 & 2 \end{bmatrix} \vec{x} \quad \vec{x}(0) = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

$$12. \vec{x}' = \begin{bmatrix} 1 & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix} \vec{x} \quad \vec{x}(0) = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$