Calculus I, MATH 2413

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Text: Calculus and Analytic Geometry, Early Transcendentals, ed 5 or 6, *James Stewart*

1	Chapter 0	Verification Helpers and Calculators
2	App 0.1	Function Analyzer (X. Gang) Evaluate, graph, differentiate, integrate, solve equations, expand into Taylor series. (Fractional powers of negative numbers should be handled carefully here.)
3	App 0.2	Wolfram Natural Language Portal
4	App 0.3	Calculator
5	App 0.4	Calculator
6	Chapter 1	PreCalculus and Preliminaries
7	Pre Req	WATMU College Algebra Course Algebraic expressions, functions, graphs.
8	Pre Req	David Joyce Trigonometry Course Angles, trig functions and their inverses, graphs, identities, laws of sine and cosine, solving a triangle.
9	Chapter 2	Limits and Derivatives
10	Scroll 1	Section 2.1, Tangent Lines and Velocity Introduction to calculus. Two branches of calculus: Differential and Integral calculus. Finding slope and equation of the tangent line to the graph of a function at a given point by a limit process. Synonyms for slope: speed, rate, gradient, slant, incline, steepness, grade, pitch. Angle of inclination or tilt angle. Finding velocity of a particle given its position as a function of time.
11	Scroll 2a	Sections 2.2, 2.3, The Limit of a Function Limits, one-sided limits, infinite limit, vertical asymptotes, The Squeeze Theorem, basic limit laws .
12	Scroll 2b	Sections 2.2, 2.3, The Limit of a Function, an Example Finding limits using a calculator through a table of values, effective use of a calculator, finite precision issues.
13	Scroll 3	Sections 2.3, The Limit of a Function, Algebraic Methods Finding limits using factorization and rationalization
14	App 3.1	Examples of limit problems done using algebra (L. S. Husch)
15	App 3.2	More examples of limit problems done using algebra (L. S. Husch)
16	Scroll 4	Sections 2.2, 2.3, Vertical Asymptotes and Infinite Limits Vertical asymptotes of rational, trigonometric and logarithmic functions.

17	Chapter 2	Limits and Derivatives
18	Scroll 5	Section 2.4, The Precise Definition of a Limit Finding limits using the epsilon-delta definition. The reason for rigorous approach to mathematics. A counter-intuitive case: Sum of a series may depend on the order of summation. Infinite limits.
19	App 5.1	This applet helps you to understand the precise definition of limits.
20	Scroll 6	Section 2.5 Continuity Continuity at a point, discontinuity, types: removable, infinite, oscillatory, one-sided continuity, continuity on an interval, continuous operations with continuous functions, continuity of basic functions, The Intermediate Value Theorem
21	App 6.1	This advanced applet helps you to construct pieces-wise defined functions by writing short programs.
22	Scroll 7	Limits at Infinity, Asymptotes The behavior of a function as x goes to positive or negative infinity, left tail and right tail, infinite limits, unbounded and bounded oscillations, horizontal asymptotes, examples from rational, root, exponential and trigonometric functions, precise definitions. Correction: At minute 32 of the video I draw y= - 2/3 instead of y=2/3 as the horizontal asymptote.
23	App 7.1	If you have a low degree rational function in a factored or standard form use the following applet. You can change the parameters of certain functions and see how asymptotes and the shape of the functions are influenced.
24	Note	In Edition 5ET Derivatives are introduced in two sections, 2.7 and 2.8. In Edition 6ET these sections are combined into one section namely 2.7.
25	Scroll 8	Review of Sections 2.1-7 A review of what we have done so far using the limits notation. Finding slopes of tangent lines using limits. Four basic examples. Finding instantaneous velocity using limits. Projectile, turning point, maximum height, time to strike.
26	Scroll 9	Section 2.7/Ed 5 or Section 2.8/Ed 6 Notation Derivatives, basic notation, equation of tangent line, applications.
27	Scroll 10	Section 2.9/Ed 5 or Section 2.8/Ed 6 Derivative as a Function Derivative as a function, Newton and Leibniz notations, operators, differential operators, differentiability, differentiable at a point, differentiable on an interval, when derivative fails to exist, sharp corners and cusps, one-sided derivatives, Theorem: differentiability implies continuity.
28	App 10.1	Choose a function from the drop down menu, or type one yourself, for example to get $5x^3 + 20\cos(\pi x) + e^{-2x}$ you type 5*x∧3+20*cos(pi*x)+e∧(-2*x), (notice that we use ∧ for power and * for multiplication, the software understands pi as π and e as the base of e^x), set boundaries for the picture and drag the horizontal slider. Try to guess what the graph is going to do and then verify it.
29	App 10.2	The trace will draw the derivative of the function.
30	App 10.3	Puzzle: Match the function to its derivative.

31	Chapter 3	Limits and Derivatives
32	Scroll 11	Section 3.1, Derivatives of Polynomial and Exponential Functions Derivative of monomials x^n, derivative of x^n for rational and negative n, linearity, derivative of polynomials, derivative of exponential functions, 2^x, 3^x, a definition of e, derivative of e^x, examples.
33	Scroll 12	Section 3.2, Product Rule and Quotient Rule of Differentiation The Product Rule, The Quotient Rule, general power rule, power-quotient rule, examples, an explanation of product rule and quotient rule
34	App 12.1	Applet illustrating the product rule.
35	App 12.2	Type your function. Be careful about missing multiplication sign and parenthesis. Choose the variable of differentiation. Press Apply button. Dx means derivative with respect to x, Dy is for y, etc. At each step applet shows which rule has been used.
36	Note	A brief Trig review is included. You may want to start with that or consult a full course for additional material alongside calculus.
37	Scroll 13	Section 3.4, The Derivatives of Trigonometric Functions Derivative formulas for sin, cos, tan, sec, csc, cot. Geometric proofs of several important trig inequalities, for $0 < x < \pi/2$, x in radians, we show (1) $\sin x < x < \tan x$, (2) $(x/2)\sin x < (1 - \cos x) < x\sin(x/2)$
38	App 13.1	Comprehensive Calculus I Applet. To experiment with just about any function and compare your results against an automatically generated solution you may use applet for differentiation, graphing, finding max and min. Experiment with the following buttons, "The Value or Limit", "f'(x)", "The curve of". You will learn about other buttons later.
39	Prereq 13.1	Trigonometry Review Angles, vertex, ray, opening, initial side, terminal side, positive or counter-clockwise direction, negative or clockwise direction, winding number, standard position, quadrants, circumference of a circle, measuring angles, degrees, radian, grad, circle, cycle, rotation, round, length of an arc, area of a sector, six trig functions in terms of adjacent, opposite, and hypotenuse, solving a right triangle, six trig functions for arbitrary angles, trig identities: Pythagorean, ratio, negative, sum of angles.
40	Prereq 13.2	David E. Joyce Trigonometry Course.
41	App 13.2	The six trig functions in a unit circle setting.
42	App 13.3	The animated drawing of sine, cosine and tangent functions. Click on the third big red box titled: "Applet: the graph of sin, cos, and tan."
43	App 13.4	Click on "Graphs of elementary trigonometric functions."

44	Chapter 3	Limits and Derivatives
45	Scroll 14	Section 3.5 ed 5, or Section 3.4 ed 6, The Chain Rule Composition of functions, the chain rule formula, examples, geometric explanation, and an indication of the simplified proof. Correction One item missing from this lecture is the derivative of a^x. Here is the formula: $(a^x)' = a^x \ln a$. (See Page 222 Edition 5/Page 201 Edition 6) Here is the proof: 1) You can use chain rule to show$[e^{bx}]' = be^{bx}$. 2) From properties of logarithm we know $a = e^{lna}$ hence $a^x = e^{xlna}$. 3) Now differentiate both sides $(a^x)' = [e^{x \ln a}]' = e^{x \ln a} \ln a = a^x \ln a$.
46	App14.1	The Chain Rule Applet 1 To experiment with a chain rule applet you can define two functions, such as $y = f(x) = 3 - (x^2)/2$ and $y = g(x) = sin(x)$, and see the graphs of f, g and $g(f(x))$, as well as the tangent lines and their slopes, at a point$(x_0, f(x_0))$ (the red spot in the left picture), point$(f(x_0), g(f(x_0)))$, on the second picture, and point $(x_0, g(f(x_0)))$, on the third picture. Notice that you can drag the red spot. What does the color-coordination of various line segments mean? How does the third slope relate to the first two?
47	App 14.2	The Chain Rule Applet 2 Check both Normal Mode and Delta Mode buttons. You can modify angle of view by pressing up/down/left/right buttons, and move on curves by pressing $x+$ and $x-$ buttons. Can you decipher the picture?
48	Scroll 15	Section 3.6 ed 5ET, or Section 3.5 ed 6ET, Implicit Differentiation derivatives of inverse trig functions, orthogonal families of functions Finding dy/dx given $F(x, y) = 0$, application to finding derivatives of inverse functions, inverse sine function arcsin or $\sin^{-1}$ as well as $\cos^{-1}, \tan^{-1}, \sec^{-1}, \csc^{-1}, \cot^{-1}$. Examples of orthogonal trajectories.
49	Scroll 16	Section 3.7 (ed 5 only), Higher Order Derivatives Higher order derivatives, position, velocity, acceleration, graphical interpretation, derivative notation, factorial, high order derivatives of $x^a, \sin(x), \cos(x)$, implicit higher order derivatives. (This section is eliminated in Edition 6. However this topic is related to Chapter 11. A short piece shows up in Section 3.3 Page 194.)
50	Scroll 17	Section 3.8 Ed 5, or Section 3.6 Ed 6, Derivatives of Logarithmic Functions Derivative of logarithm in base b, derivative of natural log, logarithmic differentiation, derivatives of various power types b^x, x^b, u^v, another description of e the base of natural logarithm.
51	App 17.1	Drills on Logarithmic Differentiation
52	Prereq 17.1	Section 1.6, Elementary description of logarithms, rules of logarithms, base 10 and base e graphs.
53	Scroll 18	Section 3.9 Ed 5, Section 3.11 Ed 6, Hyperbolic Functions Part 1 Use the text for formulas for the inverse hyperbolic functions. Definitions of sinh, cosh, tanh, csch, sech, coth. Graphs of the hyperbolic functions. Differentiation formulas. Basic identity.
54	Scroll 19	Section 3.9 Ed 5, or Section 3.11 Ed 6, Basic exercises on Hyperbolic Functions Solution of Problems 11, 32, 39, 41.
55	Scroll 20	Section 3.9 Ed 5, Section 3.9 Ed 11, Inverse Hyperbolic Functions $\sinh^{-1}$ or arcsinh or asinh and other inverses, their graphs, domains, derivatives, formulas in terms of natural log.
56	Scroll 21	Section 3.9 Ed 5, Section 3.9 Ed 11, A Geometrical View of Hyperbolic Functions, What does *cosh*, the hyperbolic cosine, have to do with hyperbola and cosine? Connections between hyperbolic and circular trigonometric functions, Euler formula.

57 Chapter 3 Limits and Derivatives

58 **Note** This section is very interesting and challenging. If you were asking why we are learning all these math formulas you get a partial answer in this section. Do read the text and spend time with the problems done in the text. Also expect a rough road and prolonged delays before you get the hang of it. If you want mathematics to be a major part of your career then you must do well in this section. We will have three sections in calculus I that are like this: related rates, graphing, and optimization.

59 **Scroll 22** Section 3.10 Ed 5, Section 3.9 Ed 6, Related rates.
Study of problems from geometry, physics, and engineering where several quantities are related to each other and we use information about their current values and current rates of change to find a missing rate of change. Solutions of three problems are presented. Read the text for introductory problems and basic advice on how to get started.

60 **Note** After you view several problems check to see how the following steps were carried out and try to apply the same general approach. Redo all problems that are done in the text and in the video.

1. Make a listing of variables whose rates of change are given or requested.
2. Draw a clear picture, if applicable.
3. Give variables names $x, y, v, ..$ and show them on the picture. Typically you differentiate with respect to time t not x.
4. Write the rates of change of variables and their current values in a table.
5. Discover the relationship between the variables. This is the hard part; it may need geometry, visualization, or basic science etc. Review geometry formulas under the front cover.
6. Differentiate this relationship with respect to time carefully. For example remember $(x^3)'$ is not $3x^2$, it is $3x^2x'$.
7. Substitute the values you know to find the one you do not know. The current values of variables do not get to be used until this final stage.

61 **App 22.1** Here you can see ten related rates demos: Overhead kite/airplane, sand pile, sliding ladder, shadow of a walking figure, oil spill, rolling snowball, elliptical trip, opening a window on a computer screen, and baseball runner. Table of links at the bottom of page.

62 **App 22.2** Demo of two ships moving with respect to each other.

63 **Scroll 23** Section 3.11 Ed 5, Section 3.10 Ed 6, Linearization and Differentials
Linear Approximation, tangent line approximation, Linearization, small angle approximation, differentials, relative error, percentage error. Solution of several sample problems.

64 **App 23.1** Linear approximation app. You can choose or type a function$f(x)$, in place of x^2, choose the base of operation, a, and the displacement, h, as in $f(a+h) \approx f(a) + hf'(a)$. The horizontal slider changes h, the vertical slider changes the slope of the line. The blue inset is the picture of the error of approximating the graph by the line you have chosen, so if your line is $y = mx + b$ the error function is $E(x) = f(x) - (mx + b)$.

65	Chapter 4	Applications of Differentiation
66	Scroll 24	Section 4.1, Maximum and Minimum Points Definition of absolute or global maximum or minimum, relative or local max or min, extremum points, critical points, The Extremum Value Theorem, Fermat's Theorem, The Closed Interval Method. Solution of problems 47, 53, 56.
67	App 24.1	Comprehensive Graphing Applet Note: variable is x. If a rational exponent is used, as in $x^{1/3}$, applet restricts domain to positive numbers.
68	App 24.2	Investigating relationship between a function, its derivative, maximum, minimum, and tangent line .
69	Scroll 25	Section 4.2, The Mean Value Theorem. Rolle's Theorem. A function with a zero derivative is a constant; functions with identical derivative differ by a constant. Solution of Problems 12, 20.
70	App 25.1	Here is an interesting applet for the Mean Value Theorem or Rolle's Theorem . Go to the middle of page; play with both "Drag curve and Drag tangent" options.
71	Scroll 26	Section 4.3, Influence of Derivatives on the Shape of a function. Increasing/Decreasing test, the first derivative test, Concavity, The second derivative test, inflection point, a sample problem.
72	Scroll 27	Section 4.4, L'Hôpital's Rule and Indeterminate forms. $0/0, \infty/\infty, 0*\infty, \infty-\infty, 1^\infty, \infty^0, 0^0$. Solution of 12, 23, 45, $\lim_{x\to\infty}(1+1/x)^x$, and other sample problems. Correction: The discussion of 0^0 case is missing from the video. The treatment is similar to ∞^0 or 1^∞ case. Please read about it in the text.
73	Note	Understanding Section 4.5 is a very good indicator of how well you have understood the entire math you have learned so far. It is a culminating point for the first half of calculus. Section 4.5 will be time consuming, challenging, and important, so plan in advance. Read the text, redo the examples in the text, view the next three videos, and, Do Problems 5, 13, 19, 21, 24, 27, 31, 41, 45, 59.
74	Scroll 28	Graphing 1 Point Plotting, Domain, Range, Intercepts, Symmetry, Asymptotes, First derivative analysis, Second derivative analysis.
75	App 28.1	Comprehensive Graphing Applet Note: variable is x. If a rational exponent is used, as in $x^{1/3}$, applet restricts domain to positive numbers.
76	Scroll 29	Section 4.5, Curve Sketching 2 Problems 4 and 18.
77	Scroll 30	Section 4.5, Curve Sketching 3. Problems 47 and 42.
78	Scroll 31	Section 4.2, 4.3, 4.4 Review Rolle's Theorem, Section 4.2, Problem 3, Shape of a graph, Section 4.3, Problems 15, 38, 40, L'Hôpital's Rule, Section 4.4, Problem 48. Correction: A couple of minutes of the video, in the middle of the third question (problem 38), is missing.

79	Chapter 4	Applications of Differentiation
80	Note	This is a very interesting section. You will see real applications here. Spend time on translating from English to Math. If you allocate time you will get the benefit in the long run. Again, if you want to have math play a serious role in your future education you need to do well in this section.
81	Scroll 32	Section 4.7, Optimization. Constraints, objective functions, Snell's Law, Fermat's Principle, Solution of a sample problem and 16, 25, 51.
82	App 32.1	A gutter with maximum area.
83	App 32.2	Rectangle with largest area inscribed in a right triangle. Similar to Problem 24.
84	App 32.3	Moving a pipe around a corner between two hallways. A bit more challenging than others. Similar to Problem 54.
85	Scroll 33	Section 4.8 Ed 6, Newton-Raphson Method Method for finding roots of equations, approximate methods, graphical explanation, algorithm, example, pathological cases. Correction: A statement made at the beginning of this lecture says there are no formulas for roots of polynomials of degree four and above. The correct description is: There are no formulas (similar to the quadratic formula) for equations of degree five and above.
86	Note	An important issue here is the efficient use of a calculator. Make sure you know how to use your own calculator for storing a number and recalling it and grouping expressions in parentheses. There are keys for these activities but each calculator has a different style. Check the calculator's manual or its website. Here is a simple test example let $x = 0.123456789$. Find $(x + x^2) * sin(x + x^3)$, but enter x *only once.*
87	App 33.1	Applet shows the successive steps of Newton's Method. Click somewhere on the X-axis.
88	App 33.2	For Your Eyes Only!, Newton Basin Fractals A polynomial has as many roots as its highest power. Depending on where you start Newton's method you will generally converge to one of the roots. This applet colors starting points according to which root they lead to. The whole operation is done on the complex plane, $z = x+iy$, and creates amazing pictures. Highly recommended for a rainy day. Some non-interactive low-resolution sample pictures are here. But it is better if you create your own and zoom on it, as that is done here. To zoom simply left-click and drag the mouse over the diagonal of the rectangular region you want to enlarge. You can find more about Newton fractals here. The newton basin fractal image for $f(z) = z^3 - 1$ is here. The basic version of fractal image for $f(z) = z^3 - 1$ is here. Each color represents set of points which will lead to the same root.
89	Scroll 34	Section 4.10 Ed 5, Section 4.9 Ed 6, Antiderivatives Basic anti-differentiation formulas, initial conditions, rectilinear motion, position, velocity, acceleration, initial position, initial velocity.
90	App 34.1	Simple exercises on antiderivatives.

91	Chapter 5	Integrals
92	Scroll 35	Section 5.1, Areas and Distances Part 1. Numerical approximation of area under a curve. Left point rule, right point rule, midpoint rule, an example, identities for sums of power, sum of squares, sum of cubes, area as a limit, exact area under a parabola.
93	App 35.1	Applet helps you see the rectangles for the Riemann sums.
94	Scroll 36	Section 5.1, Areas and Distances Part 2. Sigma notation, partition of an interval, area formulas using sigma notation, finding area under $f(x) = 2x^2 + x$ over $[1, 3]$ by using partition, sum of powers formulas, and limits, calculating distance from velocity function, Theorem: Change in position in a time interval is equal to the area under the velocity function over the time interval. Corrections: 1) At the beginning of the lecture I use the phrase "summation convention" that is incorrect and should be dropped. The correct phrase is just "sigma notation". 2) In the last few seconds of the video 4+1/3 is written as 14/3. It is of course 13/3.
95	Scroll 37	Section 5.2, The Definite Integral Riemann sum, integral sign, integrand, upper limit, lower limit, properties of the definite integral: reversal, linearity, concatenation, comparison.
96	Scroll 38	Section 5.2, Evaluating a Definite Integral. Problem 22 An important example problem putting all basic concepts of area or definite integral calculation under one roof.
97	Scroll 39	Section 5.3, Fundamental Theorem of Calculus (FTC) Integration and differentiation are inverse processes. How to use FTC to calculate areas and definite integrals in a short time.
98	App 39.1	Applet lets you manipulate a graph by hand and draw its derivative, integral, and tangent line.
99	App 39.2	Applet shows signed area under a specific function.
100	Scroll 40	Section 5.4, Indefinite Integral and the Net Change Theorem. Indefinite integral as a notation for antiderivative. Table of basic indefinite integral formulas, Problems 9, 13, 14, 15, 21. The Net Change Theorem, displacement is the definite integral of velocity.
101	Scroll 41	Section 5.5, Substitution Method Substitution method for indefinite and definite integration. Two styles for calculating definite integrals. Several examples.
102	App 41.1	Exercises on substitution method 1.
103	App 41.2	Exercises on substitution method 2.
104	Scroll 42	Review of Chapter 5. Five multi-part Problems similar to the following problems Book Edition 5ET; 9-12 page 391, 21-25 page 391, 19-40 page 403, 1-40 page 411, 1-70 page 421. (The solution of the second problem is lengthy; you may want to do that at the end.)

105	Chapter 6	Applications of Integration
106	Scroll 43	Section 6.1, Area Between Curves Area between two curves, finding top, bottom, left, and right parts, criss-crossing curves, practice with x as a function of y, general regions.
107	App 43.1	Area between curves calculated by Riemann sums.
108	Scroll 44	Section 6.2, Volumes and Method of Washers. Volume of a cylinder, volume of a solid, cross section, solids of revolution, volume by method of washers or annular rings, examples of non-rotational solids.
109	App 44.1	Applet takes you through steps of an example for method of washers.
110	App 44.2	This applet creates solids of revolution from a curve.
111	App 44.3	This applet is similar to the last problem in the video but not identical.
112		Washer Method
113	App 44.4	Visualization of slices of a wedge of a cylinder. Similar to Example 9 Page 452. Intersecting cylinders.
114	Scroll 45	Section 6.3, Volumes by Cylindrical Shells. Volume of a cylindrical shell, calculating volume by cutting a solid of revolution into cylindrical shells, examples, reason for having two different methods for calculating volume.
115	App 43.1	Applet takes you through steps of calculating a volume by shells.
116	Scroll 46	Review of Chapter 6.1-3. Three multi-part Problems similar to problems 5-26 page 442, problems on page 452, problems 3-20 page 458.
117	Scroll 47	Section 6.4, Work Definition, physical units, several examples, problems 1, 14, 15.
118	App 47.1	Several examples of calculation of work.