Print: Last Name First Name

Calculus and analytic geometry 2413, Sept 23, 2010, Test #1

Show ALL work on white pages. Please follow the mathematical writing rules.

1(10p). DOMAIN. (a) Make a sign chart for $\dfrac{x^2+9x+14}{x^2-16}$.

(b) Find the domain of $\sqrt{\dfrac{x^2+9x+14}{x^2-16}}$. Write it in interval notation.

2(12p). SLOPE BY TABLE. The point $P(5, \sqrt{5})$ lies on the curve $y = \sqrt{x}$.

(a) Consider point $Q = (x, \sqrt{x})$. Write the expression for the slope $m_{PQ}$ of the secant line $PQ$.

(b) Calculate $m_{PQ}$ to <u>eight</u> decimal places, for the following values of $x$ and complete the table below with your answer.
Hints: 1) Do NOT confuse $y$ with $m$!, 2) You may find it easier to simplify $m_{PQ}$ first, then evaluate at specific values of $x$.

| $x$ | 5.00001, | 5.0001, | 5.001, | 5.01, | 5.1 |
|---|---|---|---|---|---|
| $m_{PQ}$ | | | | | |

(c) Check the results of part (a). Which part of digits of $m_{PQ}$ has stabilized as $x$ approaches 5? Report just that portion as the best approximate value of the slope of the tangent line to the curve at $P$. (You need to follow the instruction and avoid guessing/shortcuts.)
$m \approx ?$

(d) Using the approximate slope from part (b), find an equation of the tangent line to the curve at $P$. Write it as $y = mx + b$ where $m$ and $b$ are decimal approximations.

3(14p). PROJECTILE/MECHANICS. A rock is thrown upward on some planet and its height in meters $t$ seconds later is given by $y = 20t - 2t^2$.

a) Find the exact average velocity for the time period beginning when $t = 6$ and lasting

I) .1 second

II) .01 second

III) .001 second

b) Guess the instantaneous velocity when $t = 7$.

c) Is the rock going up or down at $t = 6$?

d) Find the time at which the rock hits the ground.

4(5p). LIMITS. Find the exact value of required limits by using limit laws and <u>algebraic</u> steps (you can use table approach to double check your answer but algebraic steps are needed for full credit).

$$\lim_{x\to 2} \frac{\sqrt{x+7}-3}{x^2-4}$$

5(10p). ONE-SIDED LIMITS. Let $f(x) = \begin{cases} x^2-1 & \text{if } x<2, \\ 3x^2+x & \text{if } 2 \le x < 4, \\ 5x^2-x-24 & \text{if } 4 \le x, \end{cases}$

find each value or limit

a) $\lim_{x\to 2^-} f(x) =$ $\lim_{x\to 2^+} f(x) =$

b) $\lim_{x\to 2} f(x) =$ $f(2) =$

c) $\lim_{x\to 4^-} f(x) =$ $\lim_{x\to 4^+} f(x) =$

d) $\lim_{x\to 4} f(x) =$ $f(4) =$

e) At what points is $f$ discontinuous?

6(7p). CONTINUITY. Determine $k$ so that $f$ is a continuous function. Show your reasoning and all steps. $f(x) = \begin{cases} x^2-k & \text{if } x<2, \\ 3x^2+2k & \text{if } 2 \le x. \end{cases}$

8(10p). LIMITS AT INFINITY. Find the limit and write the equation of the corresponding asymptote.

a) $\lim_{x \to -\infty} x + \sqrt{x^2 + 5x + 1}$

b) $\lim_{x \to -\infty} \frac{\sqrt{3x^2 + 5x}}{2x + 7}$

9(10p). PRECISE DEFINITION OF LIMITS. Find the largest $\delta$ such that $\left| \frac{1}{x} - 0.25 \right| < 0.1$ whenever $|x - 4| < \delta$. Explain your steps by drawing a graph.

10(6p). ASYMPTOTES. Draw the graph of each function. Clearly specify behavior near asymptotes.

$y = \frac{-3}{2 - x}$

11(10p). DERIVATIVES. Use the limit definition to find $f'(x)$ in each case.
a) $f(x) = 2x^3 + x$

b) $f(x) = \dfrac{2x+3}{5x+7}$.

12 (6p). VISUAL DIFFERENTIATION. Graph of $f(x)$ is drawn. Carefully draw $f'(x)$ on the same axes.