Differentiate the following functions, simplify, and perform the required tasks.

a) \( f(x) = \frac{x^2 + 9x + 14}{x^2 - 16} \), find \( f'(x) \) and simplify.

b) \( f(x) = x^4 e^x \), find roots of \( f'(x) = 0 \).

c) \( f(x) = \csc(x) + e^x \cot(x) \), find \( f'(x) \).

d) \( f(x) = \frac{1 + \sin x}{x + \cos x} \), find roots of \( f'(x) = 0 \).
e) $f(x) = \cos^5(x^3 + x^2)$, find $f'(x)$.

f) $f(x) = (2x + 3)^5(4x + 5)^7$, find roots of $f'(x) = 0$.

g) $x^4y^2 + 8x^3y^3 + x^2y^4 = 10$, find $y'$ using implicit differentiation, also find equation of tangent line at (1, 1).

h) Use Logarithmic differentiation to find $f'(x)$ if $f(x) = \sin(x)^{\cos(x)}$
2) Find and simplify (using factorial notation) $f^{(100)}(x)$ and $f^{(101)}(x)$ if $f(x) = \frac{5}{x^7}$.

3) At noon, ship A is 200 km west of ship B. Ship A is sailing north at 40 km/h and ship B is ALSO sailing north at 70 km/h. How fast is the distance between ships changing at 6:00 P.M.?

4) Use differentials or linear approximation to estimate $\sqrt{999.1}$. Show all steps.
5a) What is the differential of \( y = x^3 + x^2 \).

5b) Evaluate \( dy \) for \( x = 1 \) and \( dx = 0.1 \).

5c) Evaluate \( \Delta y \) for \( x = 1 \) and \( dx = 0.1 \).

6) Find the absolute maximum and minimum of \( f(x) = x^3 + x^2 - x + 1 \) on \([-2, 1]\).

7) Verify that the function satisfies the hypothesis of the Mean Value Theorem on the given interval. (List the conditions.) Find all numbers \( c \) that satisfy the conclusion of the MVT.

\[
 f(x) = \frac{x - 1}{2x + 3}, \quad [-1, 3].
\]