Review of differentiation and integration rules from Calculus I and II for Ordinary Differential Equations, 3301

General Notation:

a, b, m, n, C are non-specific constants, independent of variables

e, π are special constants $e = 2.71828\cdots$, $\pi = 3.14159\cdots$

f, g, u, v, F are functions

$f^n(x)$ usually means $[f(x)]^n$, but $f^{-1}(x)$ usually means inverse function of f

$a(x+y)$ means a times $x+y$, but $f(x+y)$ means f evaluated at $x+y$

fg means function f times function g, but $f(g)$ means output of g is input of f

t, x, y are variables, typically t is used for time and x for position, y is position or output

$','' $ are Newton notations for first and second derivatives.

Leibnitz notations for first and second derivatives are $\frac{d}{dx}$ and $\frac{d^2}{dx^2}$ or $\frac{d}{dt}$ and $\frac{d^2}{dt^2}$

Differential of x is shown by dx or Δx or h

$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$, derivative of f shows the slope of the tangent line, rise over run, for the function $y = f(x)$ at x

General differentiation rules:

1a- Derivative of a variable with respect to itself is 1. $\frac{dt}{dt} = 1$ or $\frac{dx}{dx} = 1$.

1b- Derivative of a constant is zero.

2- Linearity rule $(af + bg)' = af' + bg'$

3- Product rule $(fg)' = f'g + fg'$

4- Quotient rule $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

5- Power rule $(f^n)' = nf^{n-1}f'$

6- Chain rule $(f(g(u)))' = f'(g(u))g'(u)u'$

7- Logarithmic rule $(f)' = [e^{\ln f}]'$

8- PPQ rule $(f^n g^m)' = f^{n-1}g^{m-1}(nf'g + mfg')$, combines power, product and quotient

9- PC rule $(f^n(g))' = nf^{n-1}(g)f'(g)g'$, combines power and chain rules

10- Golden rule: Last algebra action specifies the first differentiation rule to be used

Function-specific differentiation rules:

$(u^n)' = nu^{n-1}u'$

$(e^u)' = e^u u'$

$(\ln(u))' = \frac{u'}{u}$

$(\sin(u))' = \cos(u)u'$

$(\tan(u))' = \sec^2(u)u'$

$(\sec(u))' = \sec(u)\tan(u)u'$

$(u^v)' = u^v v' \ln u + vu^{v-1}u'$

$(a^u)' = a^u u' \ln a$

$(\log_a(u))' = \frac{u'}{u \log a}$

$(\cos(u))' = -\sin(u)u'$

$(\cot(u))' = -\csc^2(u)u'$

$(\csc(u))' = -\csc(u)\cot(u)u'$

$$(\sin^{-1}(u))' = \frac{u'}{\sqrt{1-u^2}} \qquad (\cos^{-1}(u))' = \frac{-u'}{\sqrt{1-u^2}}$$

$$(\tan^{-1}(u))' = \frac{u'}{1+u^2} \qquad (\cot^{-1}(u))' = \frac{-u'}{1+u^2}$$

$$(\sec^{-1}(u))' = \frac{u'}{u\sqrt{u^2-1}} \qquad (\csc^{-1}(u))' = \frac{-u'}{u\sqrt{u^2-1}}$$

General integration definitions and methods:

1- Indefinite integral $\int f(x)dx = F(x) + C$ means $F'(x) = f(x)$, F is antiderivative of f

2- Definite integral $\int_a^b f(x)dx = F(b) - F(a)$ is area under $y = f(x)$ from $x = a$ to $x = b$

3- Linearity $\int (af + bg)dx = a\int f dx + b\int g dx$

4a- Integration by parts $\int fg'dx = fg - \int f'g dx$

4b- Integration by parts $\int u dv = uv - \int v du$

5a- Indefinite integration by substitution $\int f(g(x))g'(x)dx = \int f(u)du$ when $u = g(x)$

5b- Definite integration by substitution $\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$ when $u = g(x)$

6- Integration by partial fraction decomposition

7- Integration by trigonometric substitution, reduction, circulation, etc

8- Study Chapter 7 of calculus text (Stewart's) for more detail

Some basic integration formulas:

$$\int u^n du = \frac{u^{n+1}}{n+1} + C,\ n \neq -1 \qquad \int \frac{du}{u} = \ln(u) + C$$

$$\int e^u du = e^u + C \qquad \int a^u du = \frac{a^u}{\ln a} + C$$

$$\int \cos(u)du = \sin(u) + C \qquad \int \sin(u)du = -\cos(u) + C$$

$$\int \sec^2(u)du = \tan(u) + C \qquad \int \csc^2(u)du = -\cot(u) + C$$

$$\int \sec(u)\tan(u)du = \sec(u) + C \qquad \int \csc(u)\cot(u)du = -\csc(u) + C$$

$$\int \tan(u)du = \ln|\sec(u)| + C \qquad \int \cot(u)du = -\ln|\csc(u)| + C$$

$$\int \frac{1}{u^2+a^2}du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C \qquad \int \frac{1}{\sqrt{a^2-u^2}}du = \sin^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{1}{u^2-a^2}du = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C \qquad \int \frac{1}{\sqrt{u^2 \pm a^2}}du = \ln|u + \sqrt{u^2 \pm a^2}| + C$$

$$\int \sec(u)du = \ln|\sec(u) + \tan(u)| + C \qquad \int \csc(u)du = \ln|\csc(u) - \cot(u)| + C$$

$$\int \sec^3(u)du = \frac{1}{2}(\sec(u)\tan(u) + \ln|\sec(u) + \tan(u)|) + C$$

$$\int \csc^3(u)du = \frac{1}{2}(-\csc(u)\cot(u) + \ln|\csc(u) - \cot(u)|) + C$$