College of Arts & Sciences
LAMAR UNIVERSITY
Department of Mathematics

Ordinary Differential Equations
Dr. Maesumi
Math 5308, Fall 2022

⋆ Homework #8
⋆ Systems of ODEs, linear constant coefficient homogeneous cases, Method of Diagonalization
⋆ Please prepare to present your solutions to class in the next meeting. Also, any question about past homework may be brought up.
⋆ Second exam is on Tuesday 18^{th}. It focuses on homework 5,6,7,8, but knowledge of prior topics is assumed.
⋆ Students on path to a Ph.D. want to practice typing up the answers and including graphs and matrices using TEX when applicable.
⋆ Install JRE and then DField and PPlane to your home computer. In the lab JRE is already installed just install DField and PPlane. Both must be functional for the exam.

We have studied three methods for solving a system of first order ODEs. (A) Conversion to a second order equation (Homework 5) (B) Linear combination of two solutions (Homework 7) (C) Diagonalization of matrices (Homework 8).
Diagonalization steps in brief are: Consider the system

$$x' = ax + by$$
$$y' = cx + dy$$

Let the eigenvalues be λ_1, λ_2 and the corresponding (column) eigenvectors v_1, v_2. Define the eigenvector matrix $V = [v_1, v_2]$ and the eigenvalue matrix $L = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$. Then $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = V \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} V^{-1}$. This is known as the Diagonalization Formula. If $f(x)$ is a function with power series expansion then we can define $f(Mt)$ as $V \begin{bmatrix} f(\lambda_1 t) & 0 \\ 0 & f(\lambda_2 t) \end{bmatrix} V^{-1}$, and the solution of the linear system can be written as $\binom{x}{y} = e^{Mt}\binom{x_0}{y_0} = V \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix} V^{-1} \binom{x_0}{y_0}$. The product is to be simplified to arrive at the final solution.

1- Solve each ODE using the diagonalization method.

Show all steps as done in the lecture. Use the initial condition $x(0) = 2, y(0) = 3$.

$$x' = -1x + 2y \qquad x' = 1x + 3y \qquad x' = 4x + 8y \qquad x' = 2x + 1y$$
$$y' = -7x + 8y \qquad y' = 3x + 9y \qquad y' = 0x - 5y \qquad y' = 2x + 1y$$
