Letter

Morley's Theorem: A Walk in the Park

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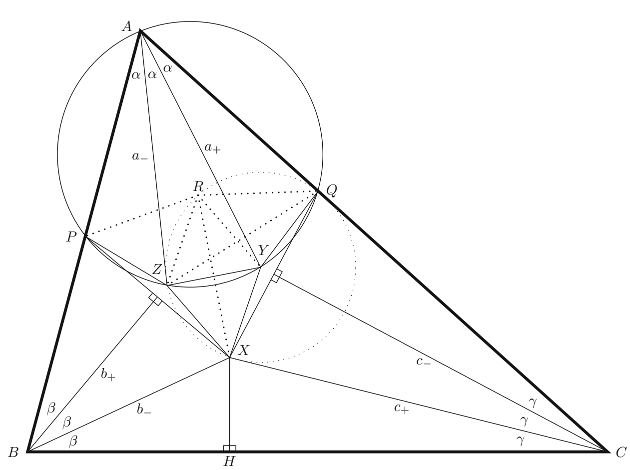
It is evident from your issues of last June and July that Morley's Theorem is once again a hot topic, and arguments are resuming as to whether Conway's delightful proof is guilty of *deus ex machina* or *lepus ex pilleo* or whatever. Conway's mischievous claim that his proof "is the indisputably simplest" dates back to 1995, and I believe it is no longer valid since my own proof below is shorter and simpler and free of any *deus* accusation. To justify this statement let's take a gentle stroll with Euclid...



Given any triangle ABC, draw and label its six internal trisectors as shown. Let X be the Morley vertex b_-c_+, and construct points P and Q on AB and AC, respectively, such that $BP = BX$ and $CQ = CX$. Let a_+ and a_- cut the circumcircle of ΔAPQ at Y and Z. The point R on the opposite side of YZ to X, such that ΔRYZ is equilateral, completes the construction.

Now for the proof itself. Evidently $PZ = ZY = YQ$ as they are chords subtending equal angles at A. Thus ΔPXR and ΔQXR are congruent since $PX = 2HX = QX$, RX is common, and $PR = QR$ by symmetry. Because the reflex angle $\angle PXQ = 2(90° - \beta) + 2(90° - \gamma) = 240° + 2\alpha$, it follows that $\angle QXR = \frac{1}{2}\angle QXP = 60° - \alpha$. The circle centre Y and radius YQ also go through R and Z, and $\angle YZQ = \angle YAQ = \alpha$ so $\angle QZR = 60° - \alpha = \angle QXR$. Therefore X too lies on this circle and thus $YX = YQ$. Hence c_- actually goes through Y so Y is the Morley vertex c_-a_+. By the same token Z lies on b_+ and therefore it is the Morley vertex a_-b_+. Finally ΔXYZ is equilateral because $XZ = PZ = ZY = YQ = YX$. ■

What's so dierent about this proof? Almost all geometric proofs rely on the prior knowledge that the Morley triangle is equilateral. They construct an equilateral triangle at an early stage and then prove that its vertices lie at the intersections of trisectors. Here this process is reversed. The trisectors appear first, and the equilateral triangle only emerges right at the very end.

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