# Linear Algebra, MATH 2318

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<th>Chapter 1</th>
<th>Introduction to Matrices</th>
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| **App 0.1** | Twist, Tumble, and Travel to your Target, a Transformation game  
A linear algebra game introducing the notion of linear transformation. Image of a home is given, you are to find a set of movements that will transform it to a target image. |
| **App 0.2** | Why do we study mathematics? An application point of view  
A list of topics in a variety of fields that are built upon mathematics, including Linear Algebra. |
| **Scroll 1** | Section 1.1, Pages 1-3, Problems 1-20  
Introduction, linear equations, a solution, solving, solution set, parametric solution, system of linear equations, linear systems, inconsistent systems, consistent systems, graphical solution, \(0/1/\infty\) Theorem: a linear system may have none or one or infinitely many solutions. |
| **App 1.1** | Interactive Online Modules for Matrix Algebra  
Investigate the graphical solution of a \(2 \times 2\) system in *The Geometer’s SketchPad*. |
| **Scroll 2** | Section 1.1, Pages 4-9, Problems 21-32  
Introduction, motivation for studying linear algebra, Gaussian elimination, triangular form, echelon form, back-substitution, coefficient matrix, augmented matrix, right hand side column, elementary operations on a linear system, elementary row operations on a matrix. |
| **App 2.1** | Linear Algebra Toolkit  
Row Operation Calculator allows you to perform or check many basic matrix operations. |
| **App 2.2** | Row Reducer  
Row Reduction Calculator performs the arithmetic of eliminations steps. |
| **App 2.2** | 3-D Plotter  
Use this interactive app to see how the graph of a linear function, e.g., \(z = 2x + 3y\), differs from a nonlinear one, e.g., \(z = x^2 \cos(y)\). |
| **Scroll 3** | Section 1.2, Pages 10-13, Problems 1-14  
Gaussian Elimination, parametric equations and deciding 0, 1, or infinitely many solutions, echelon form, leading zeros, pivot elements, leading/dependent variables, free/independent variables, how to solve an equation given in echelon matrix form. |
| **Scroll 4** | Section 1.2, Pages 13-21, Problems 15-21  
Gaussian Elimination, Elementary row operations used to reduce an equation to its echelon form; so that it can be solved by back-substitution. |
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| **Scroll 5** | Section 1.3, Pages 23-27, Problems 1-14  
Problem 15 from 1.2, matrix entry or element, matrix size or dimension, double subscript, matrix addition and subtraction, scalars and matrices, multiplication of a scalar and a matrix, dot product or inner product, matrix product as a collection of dot products. |
| **App 5.1** | Matrix Calculator  
Use this calculator to check your multiplications. |
| **App 5.2** | Interactive Online Modules for Matrix Algebra  
A visual demo of multiplication of $2 \times 2$ matrices. |
| **App 5.3** | Wolfram Demonstrations Project  
See how matrix product is constructed from the dot product of rows and columns. |
| **Scroll 6** | Section 1.3, Pages 27-29, Problems 15-16  
Matrix product as a collection of dot products, general formula using summation notation. |
| **Scroll 7** | Section 1.3, Pages 30-31, Problems 21-32  
Interpretations of matrix multiplication, multiplication by a diagonal matrix, a linear system written as matrix product, substitution as matrix multiplication, matrix multiplication as a sum of products of columns with rows. |
| **Scroll 8** | Section 1.3, Pages 32-34, Problems 33-42, Section 1.4, Pages 38-39  
Different styles for matrix multiplication, interpretations of $AB$ in terms of (a) dot products of rows of $A$ with columns of $B$, (b) matrix $A$ with columns of $B$, (c) rows of $A$ with matrix $B$, (d) row matrices of $A$ with column matrices of $B$, (e) sum of outer products of columns of $A$ with corresponding rows of $B$, and (f) $AX$, with $X$ a vector, as the linear combination of columns of $A$ with weights same as entries of vector $X$. |
| **Scroll 9** | Section 1.4, Pages 38-45, Problems 1-14  
Inverses and elementary matrices, the elementary matrix associated with multiplication of a row and exchange of two rows. |
| **Scroll 10** | Section 1.4, Pages 38-45, Problems 15-24, 36, 39, 40-43, 45, 49, 50  
Inverses and elementary matrices  
Elementary column operations (related to problem 50).  
Problem 37 from 1.3. Different styles (row expansion, column expansion, outer product expansion) for multiplication of matrices, related to lecture 8. Elementary matrix associated with adding multiple of a row to another row. General procedure for finding the inverse. |
| **Scroll 11** | Section 1.4, Pages 48-49, Problems 9-20, 25-45  
Finding Inverses using elementary matrices  
An example of $3 \times 3$ matrix being inverted.  
A short discussion of non-invertible matrices |
| **Scroll 12** | Section 1.5, Pages 57-62, Problems 1-8  
LU factorization for a simple case (without permutation), description of algorithm and justification of procedure. |
| **Scroll 13** | Section 1.5, Pages 62-64, Problems 9-24  
Solving $AX = B$ via LU factorization, advantages of LU factorization, solving systems with factorization, general permutation matrices, $PA = LU$ factorization. |
| **Scroll 14** | Section 1.5, Pages 64-67, Problems 33-36, Section 1.6 Pages 71-72, Problems 1-7, 13, 14,19, 20, Section 3.1 Pages 130-131  
Solving $AX = B$ via $PA = LU$ factorization, an example, symmetric, skew-symmetric matrices, determinants of $2 \times 2$ and $3 \times 3$ matrices, vectors in physics |
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<td>Scroll 15</td>
<td>Section 3.1, Pages 132-137, 151, Problems 1-30 with the exclusion of <em>span</em> questions. Vectors, scalars, vectors, matrices, graphical addition, subtraction, and scalar multiplication of vectors.</td>
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<td>Scroll 16</td>
<td>Section 3.1, Pages 137-141, Problems 1-30. Norm(length, size, magnitude) of a vector, distance formula, dot product of vectors, angle between vectors, length in terms of dot product, dot product in terms of components. Example: find the angle between two vectors.</td>
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<td>Section 3.1, Pages 140-144, Problems 31-42. Dot Product theorem, projections, Problems 5, 11, 36. Proof of Law of Cosines, Proof of $\mathbf{U} \cdot \mathbf{V} = u_1 v_1 + u_2 v_2 + u_3 v_3 = |\mathbf{U}| |\mathbf{V}| \cos \theta$.</td>
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<td>Section 3.1, Pages 144-146, Problems 43-50. Gram-Schmidt Process, Problem 21, producing a set of mutually perpendicular vectors $P_i$ out of an arbitrary vectors $V_i$ using projections.</td>
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<td>Scroll 19</td>
<td>Section 3.2, 3.3 Pages 149-165, Problems all of Euclidean n-Space, General Vector Spaces, Closure.</td>
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<td>Scroll 20</td>
<td>Review Sections, 1.4, 1.5, 1.6, 3.1, 3.2. Inverse, LU factorization.</td>
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<td>App 3.1</td>
<td>Interactive Online Modules for Matrix Algebra. Learn about the linear combination, span, and linear independence by following the demo under Vector Spaces (third row, under construction).</td>
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<td>Scroll 21</td>
<td>Section 3.4 Pages 165-169, Problems 1-24. Subspaces, closure with respect to vector addition and scalar multiplication, span.</td>
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<td>Scroll 22</td>
<td>Section 3.4 Pages 165-169, Problems 1-24. Examples of subspaces.</td>
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<td>Scroll 24</td>
<td>Section 3.4, Pages 173-176. Section 3.5, Pages 179-180. Subspaces, linear combinations, in $AX = B$ vector $B$ is a linear of columns of $A$ with weight factor $X$, linear dependence and independence.</td>
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<td>Section 3.5, Pages 179-180. Linear Independence, Problems 29 and 38 from 3.4.</td>
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<td>Section 3.5, Pages 180-185. Linear independence and dependence.</td>
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<td>Section 3.6, Pages 187-192, Problems 1-20. Basis, given a set of vectors how do we detect if they form a basis for a given space?</td>
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<td>Chapter 5</td>
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Determinants, brief review, short cuts, expansion/recursive formula |
| Scroll 29 | Section 5.1, Pages 326-329, Problems 21-39  
Determinants, three theorems |
| Scroll 30 | Section 5.2, Pages 331-334, Problems 1-9, 11-22 (part b only)  
Introduction to eigenvalues and eigenvectors |
| App 30.1  | Interactive Online Modules for Matrix Algebra  
Visual description of input and output vectors, eigenvalues and eigenvectors. |
| App 30.2  | Interactive Mathematics Project  
Demo 1 gives a visual description of eigenvectors and eigenvalues. |
| App 30.3  | Several videos for various cases of eigenvalues and eigenvectors. |
| App 30.4  | Wolfram Demonstrations Project  
Visual and experimental way of finding eigenvectors and eigenvalues. |
| Scroll 31 | Section 5.2, Pages 331-335, Problems 11-22 (parts a, b only)  
Eigenvalues (real, complex, double), characteristic polynomial of a matrix. |
| Scroll 32 | Section 5.2, Pages 331-335, Problems 11-22 (parts a, b, c)  
Finding eigenvectors of a matrix, matrix factorization into eigenvector matrix × eigenvalue matrix × inverse of eigenvector matrix, \( A = V \Lambda V^{-1} \). |
| Scroll 33 | Section 5.2, Pages 331-335, Problems 18-2  
Finding eigenvectors of a 3 × 3 matrix |
| Scroll 34 | Section 5.2, Pages 331-335, Problems 17-22  
Diagonalization, \( V^{-1}AV = \Lambda \), example of a 3 × 3 matrix |
| Scroll 35 | Section 5.2, Pages 331-335, Problems 17-22  
Repeated eigenvalues, finding eigenvectors and basis of eigenspaces for repeated roots of characteristic polynomial, several examples |
| Scroll 36 | Section 5.2, Pages 338-339., Problems 11-22 (part f),  
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Sum of eigenvalues=trace, product of eigenvalues=determinant, functions of a matrix, \( f(A) = V f(\Lambda) V^{-1} \), diagonalizability. |
| Scroll 37 | Section 5.3, Pages 342-349, Problems 1-26  
Diagonalization, non-diagonalizable matrices |
| Scroll 38 | Section 5.4, Pages 352-358, Problems 1-9  
Symmetric Matrices, properties, orthogonal (orthonormal) matrices, Theorem: Eigenvalues of a real symmetric 2 × 2 matrix are real. |
| Scroll 39 | Section 5.4, Pages 352-358, Problems 1-9  
Review of symmetric matrices, Problem 1 |
| Scroll 40 | Sections 5.2, 5.3, 5.6  
Eigenvalues, eigenvectors, diagonalization, a system of differential equations |
| Scroll 41 | Sections 5.5, Pages 352-358  
Markov processes |